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METHOD OF THE CONTROLLED CURRENT REGULATION – CALCULATION OF PARTIAL CONSTANTS k_{EM}, k_{EN}, k_{BM} AND k_{BN} FOR ELLIPTICAL ELECTRODES OF MICROLATEROLOG

METODA KONTROLOVANÉ REGULACE PROUDU – VÝPOČET DÍLČÍCH KONSTANT k_{EM}, k_{EN}, k_{BM} , k_{BN} PRO ELIPTICKÉ ELEKTRODY MIKROLATEROLOGU

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Abstract

It is about micro-well-logging of resistivity in the wall of borehole close to the wall. The well-logging uses the focused electric current; the current contours penetrate perpendicularly into the borehole wall. The array of the micro-electrodes can be not only circular, but too elliptical. The current electrode then looks like very thin elliptical/circular contour. The aim of this paper is derivation of formulas needed for calculation of partial constants denoted as k_{EM}, k_{EN}, k_{BM} and k_{BN}. The elliptical electrode array is concentric and is formed by potential electrode M/N shaped as an elliptical annulus and the current electrode E/B having too formed as an elliptical annulus. For 4-electrode Microlaterolog is on the electrode pad only the guard current electrode E, whereas, 5-electrode Microlaterolog has moreover the feeding current electrode B.

Abstrakt

Jedná se o karotážní mikroměření měrného elektrického odporu na stěně vrtu v blízkém okolí stěny vrtu. Měření používá fokusaci elektrického proudu, kdy isolinie proudu vstupují kolmo do stěny vrtu. Mikroelektrodové uspořádání může být nejen kruhové, ale i eliptické. Proudová elektroda potom vypadá jako eliptická/kruhová křivka. Cílem této práce je odvození vzorců potřebných pro výpočet dílčích konstant označených jako k_{EM}, k_{EN} , k_{BM} and k_{BN}. Eliptické elektrodové uspořádání je koncentrické a je tvořeno potenciálovou elektrodou M/N ve tvaru eliptického mezikruží a proudovou elektrodou E/B, která má také tvar eliptického mezikruží. Pro 4elektrodový Mikrolaterolog je na elektrodové podložce jen stínící proudová elektroda E, zatímco pro 5elektrodový Mikrolaterolog je navíc i napájecí elektroda B.

Keywords

elliptical electrode array, ellipsis, annulus, partial constants, Microlaterolog, well-logging

Klíčová slova

eliptické uspořádání elektrod, elipsa, mezikruží, dílčí konstanty, Mikrolaterolog, karotáž

1 Introduction

For derivation of the partial constants it is possible to proceed by classical way how it is depicted on fig.1. The figure supposes double integration over plain, because fig.1 presents two single Cartesian systems. The first is denoted as (x, y), the second is system (h, k).The traditional mathematical process you can accelerate, because you can profit the before derived formula. I had the before derived formula for electrode array when the current electrode is formed like annulus surrounding the potential electrode like ellipsis. This formula will be applied for deduction of next new formulas. Potential of annulus can be determined as difference of two potentials belonging two concentric ellipses. This way is simpler and faster. I start from formula derived in paper RYŠAVÝ, F. (2016) where the current electrode is annulus and the potential electrode is ellipsis.

Syntax of the method of the controlled current regulation is based on two fundamental formulas. The first solves calculation of the main constant of the electrode array denoted as K; the second solves calculation of the coefficient of focusing denoted as η. On condition of regulation that holds $U_N = U_M$ those formulas have following form:

$$K = \left\{ \left(k_{AM}^{-1} - k_{BM}^{-1} \right) + k_{EM}^{-1} \times \eta \right\}^{-1} \text{and} \quad \eta = \left(\frac{k_{AN}^{-1} - k_{AM}^{-1}}{k_{EM}^{-1} - k_{EN}^{-1}} \right) + \left(\frac{k_{BM}^{-1} - k_{BN}^{-1}}{k_{EM}^{-1} - k_{EN}^{-1}} \right).$$

The formulas are identical to those for Laterolog, RYŠAVÝ, F (2013). All variables are the partial constants denoted with indexes determining the current and potential electrodes. The central potential electrode has shape as elliptical/circular annulus and is denoted like M/N. The current electrode is elliptical/circular annulus denoted as E/B. This electrode array is characteristic for the 4-electrode Microlaterolog, possibly, for the 5-electrode Microlaterolog.

It is evident that important are partial constants; k_{EM} / k_{EN}, however too, k_{BM} / k_{BN} when it is the 5-electrode Microlaterolog. As derivation of final formulas for k_{AM} and k_{AN} is topic of other paper, the main aim of this paper is derivation calculation of the partial constant denoted as k_{EM}, k_{EN} , k_{BM} and k_{BN}. The present work is oriented only and only to calculation of these partial constants. The detail complex analyse for Microlaterolog I am going to publish in a next paper.

As you will derive the final formulas for elliptical electrode array you are able to receive the formulas for circular electrode array too. You can only imply condition that both half-axes of ellipses are equal. So thanks to the elliptical electrode array you can very easy go over to the circular electrode one. This is a mediated way how to get the final formulas for circular electrodes. The next independent way is to derive those formulas for circular electrode array in the direct way. In case you have the final formulas for the circular electrodes the same for both ways you receive big probability that you have them rightly derived.

2 Basic metrological characteristics

Fig.1 presents the basic metrological characteristics used in this paper. They are these:

A…the shorter half-axis of ellipsis for the potential electrode [m],

a… the shorter half-axis of the inner ellipsis of the current annulus [m],

B…the longer half-axis of ellipsis for the potential electrode [m],
b… the longer half-axis of the inner ellipsis of the current annulus [m] and
H…the annulus width of the guard electrode E/the feeding electrode B[m] and
L…the annulus width of the potential electrodes M/N[m].

Further, in fig.1 there are denoted the feeding current electrode B, and the guard current electrode E. It presents usual terminology of the current electrodes in focused well-logging methods. Both next electrodes M/N are the potential ones.

3 Principles of deduction

The before mentioned formula, RYŠAVÝ, F (2016), when the current electrode is annulus and the potential electrode is ellipsis has this form:

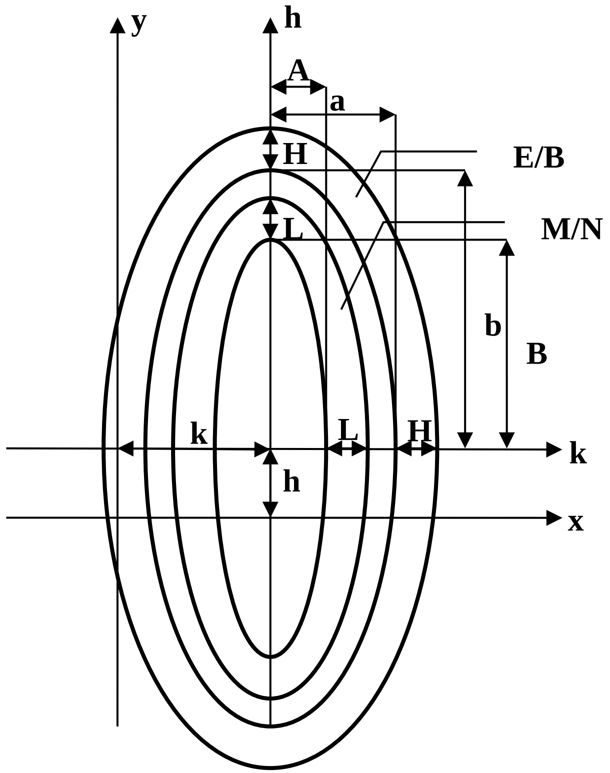

Fig.1 Depiction of two annuli like array of elliptical electrodes of Micro-laterolog

$$U^{*}=\frac{1}{2\pi}\times\left(\frac{R\times I}{A}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$
$$\times\Bigg\{+2\times\left(\frac{A}{B}\right)^{2}\times\left(\frac{a}{b}\right)^{-1}\times\Bigg[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}- \\ -\left(\frac{H}{A}+\frac{a}{A}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\Bigg] \tag{1}$$
$$+\left(\frac{a}{b}\right)\times\Bigg[\left(\frac{H}{B}+\frac{b}{B}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}+\frac{b}{B}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}-1\right)^{-1}\right\}\Bigg]\Bigg]\Bigg\}.$$

Let's return to formula (1). As we know that the voltage is decreasing with decreasing dimensions of ellipsis, it is possible the voltage of annulus to express like difference of voltages of two ellipses. Each of ellipses has different half-axes A_1, B_1 and A_2, B_2. You can define voltages for each of two concentric elliptic electrodes. The voltages are denoted as U_2 and U_1.

$$U_2=\frac{1}{2\pi}\times\left(\frac{R\times I}{A_2}\right)\times\frac{\left(\frac{H}{A_2}\right)\times\left(\frac{H}{A_2}+\frac{a}{A_2}\right)}{\left(\frac{H}{A_2}+\frac{a}{A_2}\right)\times\left(\frac{H}{B_2}+\frac{b}{B_2}\right)-\left(\frac{a}{A_2}\right)\times\left(\frac{b}{B_2}\right)}\times$$

$$\times\left\{+2\times\left(\frac{A_2}{B_2}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\left[\left(\frac{H}{A_2}+\frac{a}{A_2}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A_2}{B_2}\right)^{-1}\times\left(\frac{H}{A_2}+\frac{a}{A_2}+1\right)^{-1}\right\}-\left(\frac{H}{A_2}+\frac{a}{A_2}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A_2}{B_2}\right)^{-1}\times\left(\frac{H}{A_2}+\frac{a}{A_2}-1\right)^{-1}\right\}\right]\right.$$

$$\left.+\left(\frac{a}{b}\right)\times\left[\left(\frac{H}{B_2}+\frac{b}{B_2}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B_2}+\frac{b}{B_2}+1\right)^{-1}\right\}-\left(\frac{H}{B_2}+\frac{b}{B_2}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B_2}+\frac{b}{B_2}-1\right)^{-1}\right\}\right]\right\}. \tag{2}$$

$$U_1=\frac{1}{2\pi}\times\left(\frac{R\times I}{A_1}\right)\times\frac{\left(\frac{H}{A_1}\right)\times\left(\frac{H}{A_1}+\frac{a}{A_1}\right)}{\left(\frac{H}{A_1}+\frac{a}{A_1}\right)\times\left(\frac{H}{B_1}+\frac{b}{B_1}\right)-\left(\frac{a}{A_1}\right)\times\left(\frac{b}{B_1}\right)}\times$$

$$\times\left\{+2\times\left(\frac{A_1}{B_1}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\left[\left(\frac{H}{A_1}+\frac{a}{A_1}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A_1}{B_1}\right)^{-1}\times\left(\frac{H}{A_1}+\frac{a}{A_1}+1\right)^{-1}\right\}-\left(\frac{H}{A_1}+\frac{a}{A_1}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A_1}{B_1}\right)^{-1}\times\left(\frac{H}{A_1}+\frac{a}{A_1}-1\right)^{-1}\right\}\right]\right. \tag{3}$$

$$\left.+\left(\frac{a}{b}\right)\times\left[\left(\frac{H}{B_1}+\frac{b}{B_1}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B_1}+\frac{b}{B_1}+1\right)^{-1}\right\}-\left(\frac{H}{B_1}+\frac{b}{B_1}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B_1}+\frac{b}{B_1}-1\right)^{-1}\right\}\right]\right\}.$$

For half-axes A_1, B_1 and A_2, B_2 it holds that $A_2 > A_1$ and $B_2 > B_1$. Consequence of that is similar inequality for voltages: $U_2 > U_1$. Further, we have to define the mentioned half-axes:

$A_2 = A + L$ and $B_2 = B + L$, and (4)

$A_1 = A$ and $B_1 = B$. (5)

H presents the width of outer annulus being the current electrode, whereas, L is the width of inner annulus being the potential electrode. As it holds that a > A and b > B we must expect the following inequality:

$A + L < a$ and $B + L < b$. (6)

The voltage being on the surface of the potential annulus is expressed like difference of voltages:

$U = U_2 - U_1$. (7)

By substitution relations (4) and (5) into formulas (2) and (3) and due to relation (7) we are able to express voltage U and consequently then formula for counting of the partial constant.

$$\left(\frac{k}{A}\right) = \frac{2\pi}{\sum_{i=1}^{4} F_i}, \tag{8}$$

Constant is dependent on four functions denoted as F_1, F_2, F_3 and F_4. These functions are defined only by geometry of electrodes.

$$F_1 = +2\times\left(\frac{A}{B}\right)^{2}\times\left(\frac{a}{b}\right)^{-1}\times\left(\frac{L}{B}+1\right)^{-1}\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$
$$\times\left[\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{L}{B}+1}{\frac{L}{A}+1}\right)\times\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}+1\right)^{-1}\right\}-\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{L}{B}+1}{\frac{L}{A}+1}\right)\times\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}-1\right)^{-1}\right\}\right], \tag{9}$$

$$F_2 = +\left(\frac{a}{b}\right)\times\left(\frac{L}{A}+1\right)^{-2}\times\left(\frac{L}{B}+1\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left[\left(\frac{\frac{H}{B}+\frac{b}{B}}{\frac{L}{B}+1}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{\frac{H}{B}+\frac{b}{B}}{\frac{L}{B}+1}+1\right)^{-1}\right\}-\left(\frac{\frac{H}{B}+\frac{b}{B}}{\frac{L}{B}+1}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{\frac{H}{B}+\frac{b}{B}}{\frac{L}{B}+1}-1\right)^{-1}\right\}\right], \tag{10}$$

$$F_3 = -2\times\left(\frac{A}{B}\right)\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$
$$\times\left[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\right] \tag{11}$$

$$F_4 = -\left(\frac{a}{b}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left[\left(\frac{H}{B}+\frac{b}{B}+1\right)\times\text{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}+\frac{b}{B}-1\right)\times\text{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}-1\right)^{-1}\right\}\right]. \quad (12)$$

These are formulas needed for counting of voltage on the surface of the potential annulus. After them it is possible to enumerate the partial constants k_{EM}, k_{EN}, k_{BM} and k_{BN}. $a = b$, $A = B$.

4 Control – deduction of formula for circular annulus

Now you can do next control of calculation with the help of formulas from (9) up to (12). You do need to know the derived formulas for circular electrodes. At first we imply condition that B = A and b = a. You receive these formulas:

$$F_1 = +2\times\left(\frac{L}{A}+1\right)^{-1}\times\frac{\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+2\times\frac{a}{A}\right)}\times\left[\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}+1\right)\times\text{Argsinh}\left\{\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}+1\right)^{-1}\right\}-\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}-1\right)\times\text{Argsinh}\left\{\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}-1\right)^{-1}\right\}\right],$$

$$F_2 = +\left(\frac{L}{A}+1\right)^{-1}\times\frac{\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+2\times\frac{a}{A}\right)}\times\left[\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}+1\right)\times\text{Argsinh}\left\{\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}+1\right)^{-1}\right\}-\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}-1\right)\times\text{Argsinh}\left\{\left(\frac{\frac{H}{A}+\frac{a}{A}}{\frac{L}{A}+1}-1\right)^{-1}\right\}\right],$$

$$F_3 = -2\times\left(\frac{L}{A}+1\right)^{-1}\times\frac{\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+2\times\frac{a}{A}\right)}\times\left[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times\text{Argsinh}\left\{\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times\text{Argsinh}\left\{\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\right] \text{ and}$$

$$F_4 = -\left(\frac{L}{A}+1\right)^{-1}\times\frac{\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+2\times\frac{a}{A}\right)}\times\left[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times\text{Argsinh}\left\{\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times\text{Argsinh}\left\{\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\right].$$

$$\left(\frac{k}{A}\right)=\frac{2\pi}{\sum_{i=1}^{4}F_i}. \tag{13}$$

If you use now relation that A = D/2 and a = d/2 where both factors **D** and **d** are diameters of inside of the circular annuli, you will get following relations:

$$\left(\frac{k}{\frac{D}{2}}\right)=\frac{2k}{D}\Rightarrow\frac{k}{D}=\frac{2\pi}{2\times\sum_{i=1}^{4}F_i}=\frac{2\pi}{2\times(F_1+F_2)+2\times(F_3+F_4)}=\frac{2\pi}{G_1+G_2}, \tag{14}$$

$$G_1=+6\times\left(\frac{2L}{D}+1\right)^{-1}\times\frac{\left(\frac{2H}{D}+\frac{d}{D}\right)}{\left(\frac{2H}{D}+2\times\frac{d}{D}\right)}\times\left[\left(\frac{\frac{2H}{D}+\frac{d}{D}}{\frac{2L}{D}+1}+1\right)\times\operatorname{Argsinh}\left\{\left(\frac{\frac{2H}{D}+\frac{d}{D}}{\frac{2L}{D}+1}+1\right)^{-1}\right\}-\left(\frac{\frac{2H}{D}+\frac{d}{D}}{\frac{2L}{D}+1}-1\right)\times\operatorname{Argsinh}\left\{\left(\frac{\frac{2H}{D}+\frac{d}{D}}{\frac{2L}{D}+1}-1\right)^{-1}\right\}\right], \tag{15}$$

$$G_2=-6\times\left(\frac{2L}{D}+1\right)^{-1}\times\frac{\left(\frac{2H}{D}+\frac{d}{D}\right)}{\left(\frac{2H}{D}+2\times\frac{d}{D}\right)}\times\left[\left(\frac{2H}{D}+\frac{d}{D}+1\right)\times\operatorname{Argsinh}\left\{\left(\frac{2H}{D}+\frac{d}{D}+1\right)^{-1}\right\}-\left(\frac{2H}{D}+\frac{d}{D}-1\right)\times\operatorname{Argsinh}\left\{\left(\frac{2H}{D}+\frac{d}{D}-1\right)^{-1}\right\}\right]. \tag{16}$$

where D…the diameter of the inner circular potential annulus [m] and
d…the diameter of the outer circular current annulus [m].

The formula holds for D < d. This is formula derived for the circular annulus. It is partial case of the elliptical one. As I made, for my own control, the direct way of derivation of final formula for circular electrodes too, I can say the formula is derived rightly. Consequence of that is both formulas for circular and elliptical electrodes are rightly derived.

5 Analysis of the derived formulas

This chapter is about optimum electrode dimensions. It is about whether both electrodes ought to be wide or narrow, possibly whether the one ought to be wide and the second to be narrow and which of them. **The current annulus** can exist in two fundamental positions. The first is big width of annulus, i.e., H→∞, the second is very narrow width of annulus when holds H→0. It needs to return to the before derived formulas (9), (10), (11), (12).

You will work with the following inequalities:

$$0 \langle \left(\frac{H}{A}\right) \langle \infty \quad \text{and} \quad 0 \langle \left(\frac{H}{B}\right) \langle \infty \; ; \; 0 \langle \left(\frac{L}{A}\right) \langle \left(\frac{a}{A}\right) - 1 \quad \text{and} \quad 0 \langle \left(\frac{L}{B}\right) \langle \left(\frac{b}{B}\right) - 1 .$$

5.1 Wide current annulus

In such case you can implement into formulas (9), (10), (11), (12) condition that:

$$\left(\frac{H}{A} + \frac{a}{A}\right) \to \left(\frac{H}{A}\right) \to \infty \quad \text{and} \quad \left(\frac{H}{B} + \frac{b}{B}\right) \to \left(\frac{H}{B}\right) \to \infty .$$

For functions you receive these formulas:

$$F_1 = +2 \times \left(\frac{A}{B}\right)^2 \times \left(\frac{a}{b}\right)^{-1} \times \left(\frac{L}{B} + 1\right)^{-1} \times \frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right) \times \left(\frac{H}{B}\right) - \left(\frac{a}{A}\right) \times \left(\frac{b}{B}\right)} \times$$

$$\times \left[\left(\frac{\frac{H}{A}}{\frac{L}{A} + 1} + 1 \right) \times \text{Argsinh} \left\{ \left(\frac{A}{B}\right)^{-1} \times \left(\frac{\frac{L}{B} + 1}{\frac{L}{A} + 1} \right) \times \left(\frac{\frac{H}{A}}{\frac{L}{A} + 1} + 1 \right)^{-1} \right\} - \left(\frac{\frac{H}{A}}{\frac{L}{A} + 1} - 1 \right) \times \text{Argsinh} \left\{ \left(\frac{A}{B}\right)^{-1} \times \left(\frac{\frac{L}{B} + 1}{\frac{L}{A} + 1} \right) \times \left(\frac{\frac{H}{A}}{\frac{L}{A} + 1} - 1 \right)^{-1} \right\} \right],$$

$$F_2 = +\left(\frac{a}{b}\right) \times \left(\frac{L}{A} + 1\right)^{-2} \times \left(\frac{L}{B} + 1\right) \times \frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right) \times \left(\frac{H}{B}\right) - \left(\frac{a}{A}\right) \times \left(\frac{b}{B}\right)} \times \left[\left(\frac{\frac{H}{B}}{\frac{L}{B} + 1} + 1 \right) \times \text{Argsinh} \left\{ \left(\frac{\frac{H}{B}}{\frac{L}{B} + 1} + 1 \right)^{-1} \right\} - \left(\frac{\frac{H}{B}}{\frac{L}{B} + 1} - 1 \right) \times \text{Argsinh} \left\{ \left(\frac{\frac{H}{B}}{\frac{L}{B} + 1} - 1 \right)^{-1} \right\} \right],$$

$$F_3 = -2 \times \left(\frac{A}{B}\right)^2 \times \left(\frac{a}{b}\right)^{-1} \times \frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right) \times \left(\frac{H}{B}\right) - \left(\frac{a}{A}\right) \times \left(\frac{b}{B}\right)} \times \left[\left(\frac{H}{A} + 1\right) \times \text{Argsinh} \left\{ \left(\frac{A}{B}\right)^{-1} \times \left(\frac{H}{A} + 1\right)^{-1} \right\} - \left(\frac{H}{A} - 1\right) \times \text{Argsinh} \left\{ \left(\frac{A}{B}\right)^{-1} \times \left(\frac{H}{A} - 1\right)^{-1} \right\} \right] \text{and}$$

$$F_4 = -\left(\frac{a}{b}\right) \times \frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right) \times \left(\frac{H}{B}\right) - \left(\frac{a}{A}\right) \times \left(\frac{b}{B}\right)} \times \left[\left(\frac{H}{B} + 1\right) \times \text{Argsinh} \left\{ \left(\frac{H}{B} + 1\right)^{-1} \right\} - \left(\frac{H}{B} - 1\right) \times \text{Argsinh} \left\{ \left(\frac{H}{B} - 1\right)^{-1} \right\} \right].$$

As it holds that $(H/A)\to\infty$ and $(H/B)\to\infty$ it is possible to apply condition:

$$\left\{\left(\frac{H}{A}\right)\times\left(\frac{H}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)\right\}\to\left(\frac{H}{A}\right)\times\left(\frac{H}{B}\right).$$

Thanks to that you will get simpler expression.

$$\frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right)\times\left(\frac{H}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}=\frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right)\times\left(\frac{H}{B}\right)}=\frac{\left(\frac{H}{A}\right)}{\left(\frac{H}{B}\right)}=\left(\frac{A}{B}\right)^{-1}.$$

The simpler formulas are as follows:

$$F_1=+2\times\left(\frac{a}{b}\right)^{-1}\times\left(\frac{A}{B}\right)\times\left(\frac{L}{B}+1\right)^{-1}\times\left[\left(\frac{\frac{H}{A}+1}{\frac{L}{A}+1}\right)\times\operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{L}{B}+1}{\frac{L}{A}+1}\right)\times\left(\frac{\frac{H}{A}+1}{\frac{L}{A}+1}\right)^{-1}\right\}-\left(\frac{\frac{H}{A}-1}{\frac{L}{A}+1}\right)\times\operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{L}{B}+1}{\frac{L}{A}+1}\right)\times\left(\frac{\frac{H}{A}-1}{\frac{L}{A}+1}\right)^{-1}\right\}\right], \tag{17}$$

$$F_2=+\left(\frac{a}{b}\right)\times\left(\frac{A}{B}\right)^{-1}\times\left(\frac{L}{A}+1\right)^{-2}\times\left(\frac{L}{B}+1\right)\times\left[\left(\frac{\frac{H}{B}+1}{\frac{L}{B}+1}\right)\times\operatorname{Argsinh}\left\{\left(\frac{\frac{H}{B}+1}{\frac{L}{B}+1}\right)^{-1}\right\}-\left(\frac{\frac{H}{B}-1}{\frac{L}{B}+1}\right)\times\operatorname{Argsinh}\left\{\left(\frac{\frac{H}{B}-1}{\frac{L}{B}+1}\right)^{-1}\right\}\right], \tag{18}$$

$$F_3=-2\times\left(\frac{a}{b}\right)^{-1}\times\left(\frac{A}{B}\right)\times\left[\left(\frac{H}{A}+1\right)\times\operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}-1\right)\times\operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}-1\right)^{-1}\right\}\right] \text{ and} \tag{19}$$

$$F_4=-\left(\frac{a}{b}\right)\times\left(\frac{A}{B}\right)^{-1}\times\left[\left(\frac{H}{B}+1\right)\times\operatorname{Argsinh}\left\{\left(\frac{H}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}-1\right)\times\operatorname{Argsinh}\left\{\left(\frac{H}{B}-1\right)^{-1}\right\}\right]. \tag{20}$$

As it holds that $(H/A)\to\infty$ and $(H/B)\to\infty$ we imply conditions:

$$\left(\frac{\frac{H}{A}}{\frac{L}{A}+1}\pm 1\right)\to\left(\frac{H}{A}\right)\times\left(\frac{L}{A}+1\right)^{-1} \text{ and } \left(\frac{\frac{H}{B}}{\frac{L}{B}+1}\pm 1\right)\to\left(\frac{H}{B}\right)\times\left(\frac{L}{B}+1\right)^{-1}, \quad \left(\frac{H}{A}\pm 1\right)\to\left(\frac{H}{A}\right) \text{ and } \left(\frac{H}{B}\pm 1\right)\to\left(\frac{H}{B}\right).$$

In such case you receive the following formulas:

$$F_1=+2\times\left(\frac{a}{b}\right)^{-1}\times\left(\frac{A}{B}\right)\times\left(\frac{L}{B}+1\right)^{-1}\times$$
$$\times\left[\left(\frac{H}{A}\right)\times\left(\frac{L}{A}+1\right)^{-1}\times \text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}\right)^{-1}\times\left(\frac{L}{B}+1\right)\right\}-\left(\frac{H}{A}\right)\times\left(\frac{L}{A}+1\right)^{-1}\times \text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}\right)^{-1}\times\left(\frac{L}{B}+1\right)\right\}\right]=0, \tag{21}$$

$$F_2=+\left(\frac{a}{b}\right)\times\left(\frac{A}{B}\right)^{-1}\times\left(\frac{L}{A}+1\right)^{-2}\times\left(\frac{L}{B}+1\right)\times\left[\left(\frac{H}{B}\right)\times\left(\frac{L}{B}+1\right)^{-1}\times \text{Argsinh}\left\{\left(\frac{H}{B}\right)^{-1}\times\left(\frac{L}{B}+1\right)\right\}-\left(\frac{H}{B}\right)\times\left(\frac{L}{B}+1\right)^{-1}\times \text{Argsinh}\left\{\left(\frac{H}{B}\right)^{-1}\times\left(\frac{L}{B}+1\right)\right\}\right]=0\ , \tag{22}$$

$$F_3=-2\times\left(\frac{a}{b}\right)^{-1}\times\left(\frac{A}{B}\right)\times\left[\left(\frac{H}{A}\right)\times \text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}\right)^{-1}\right\}-\left(\frac{H}{A}\right)\times \text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}\right)^{-1}\right\}\right]=0 \text{ and} \tag{23}$$

$$F_4=-\left(\frac{a}{b}\right)\times\left(\frac{A}{B}\right)^{-1}\times\left[\left(\frac{H}{B}\right)\times \text{Argsinh}\left\{\left(\frac{H}{B}\right)^{-1}\right\}-\left(\frac{H}{B}\right)\times \text{Argsinh}\left\{\left(\frac{H}{B}\right)^{-1}\right\}\right]=0. \tag{24}$$

It is clear that it holds the sum (F_1 + F_2 + F_3 + F_4) = 0. The partial constant tends to infinity. It depends only and only on the wide current annulus. The potential annulus can be narrow or wide and does not influence this reality. Dimensions of the potential annulus in such case do not play simply any role. For wide current annulus generally holds that $k\to\infty$ and therefore $k^{-1}=0$; that is why the voltage is always zero. This variance is not acceptable.

5.2 Narrow current annulus

For such case it holds the following conditions:

$\left(\frac{H}{A}\right)\to 0$ and $\left(\frac{H}{B}\right)\to 0$.

We apply it on the before derived formulas. However, as the first you have to count limit of expression:

$$\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}$$

As it is an expression of type 0/0 you can use L´Hospital rule.

$$\lim_{H\to 0}\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}=\left(\frac{1}{A}\right)\times\lim_{H\to 0}\frac{\left(\frac{H}{A}+\frac{a}{A}\right)+\left(\frac{H}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}=\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}.$$

By adjustment of formulas (9), (10), (11), (12) you will retain these relations:

$$F_1=+2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\left(\frac{L}{B}+1\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times$$

$$\times\left[\left(\frac{\frac{a}{A}}{\frac{L}{A}+1}+1\right)\times\operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{L}{B}+1}{\frac{L}{A}+1}\right)\times\left(\frac{\frac{a}{A}}{\frac{L}{A}+1}+1\right)^{-1}\right\}-\left(\frac{\frac{a}{A}}{\frac{L}{A}+1}-1\right)\times\operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{L}{B}+1}{\frac{L}{A}+1}\right)\times\left(\frac{\frac{a}{A}}{\frac{L}{A}+1}-1\right)^{-1}\right\}\right],\tag{25}$$

$$F_2=+\left(\frac{a}{b}\right)\times\left(\frac{L}{A}+1\right)^{-2}\times\left(\frac{L}{B}+1\right)\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{\frac{b}{B}}{\frac{L}{B}+1}+1\right)\times\operatorname{Argsinh}\left(\frac{\frac{b}{B}}{\frac{L}{B}+1}+1\right)^{-1}-\left(\frac{\frac{b}{B}}{\frac{L}{B}+1}-1\right)\times\operatorname{Argsinh}\left(\frac{\frac{b}{B}}{\frac{L}{B}+1}-1\right)^{-1}\right],\tag{26}$$

$$F_3 = -2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{a}{A}+1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{a}{A}-1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}-1\right)^{-1}\right\}\right] \text{ and} \tag{27}$$

$$F_4 = -\left(\frac{a}{b}\right)\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{b}{B}+1\right)\times\text{Argsinh}\left(\frac{b}{B}+1\right)^{-1}-\left(\frac{b}{B}-1\right)\times\text{Argsinh}\left(\frac{b}{B}-1\right)^{-1}\right]. \tag{28}$$

Now, all depends only and only on the width of potential annulus. We work with inequalities:

$$0\left\langle\left(\frac{L}{A}\right)\right\langle\left(\frac{a}{A}\right)-1 \quad \text{and} \quad 0\left\langle\left(\frac{L}{B}\right)\right\langle\left(\frac{b}{B}\right)-1.$$

Narrow potential annulus

We use conditions that

$$\left(\frac{L}{A}\right)\to 0 \text{ and} \left(\frac{L}{B}\right)\to 0.$$

$$F_1 = +2\times\left(\frac{A}{B}\right)^2\times\frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{a}{A}+1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{a}{A}-1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}-1\right)^{-1}\right\}\right], \tag{29}$$

$$F_2 = +\left(\frac{a}{b}\right)^2\times\frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{b}{B}+1\right)\times\text{Argsinh}\left\{\left(\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{b}{B}-1\right)\times\text{Argsinh}\left\{\left(\frac{b}{B}-1\right)^{-1}\right\}\right], \tag{30}$$

$$F_3 = -2\times\left(\frac{A}{B}\right)^2 \times \frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)} \times \left[\left(\frac{a}{A}+1\right)\times \operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}+1\right)^{-1}\right\} - \left(\frac{a}{A}-1\right)\times \operatorname{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}-1\right)^{-1}\right\}\right] \text{ and} \tag{31}$$

$$F_4 = -\left(\frac{a}{b}\right)^2 \times \frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)} \times \left[\left(\frac{b}{B}+1\right)\times \operatorname{Argsinh}\left\{\left(\frac{b}{B}+1\right)^{-1}\right\} - \left(\frac{b}{B}-1\right)\times \operatorname{Argsinh}\left\{\left(\frac{b}{B}-1\right)^{-1}\right\}\right]. \tag{32}$$

Also this case is unpleasant, because it holds that the sum ($F_1 + F_2 + F_3 + F_4$) = 0. It is induced with fact that ($F_1 + F_3$) = 0 and ($F_2 + F_4$) = 0 too. For narrow current annulus and narrow potential annulus together holds that $k\to\infty$ and $k^{-1} = 0$. The voltage goes to zero. The narrow potential annulus is also not the optimal variance.

Wide potential annulus

It is about conditions:

$$\left(\frac{L}{A}\right) \to \left(\frac{a}{A}\right) - 1 \quad \text{and} \quad \left(\frac{L}{B}\right) \to \left(\frac{b}{B}\right) - 1.$$

$$F_1 = +4\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1} \times \frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)} \times \operatorname{Argsinh}\left\{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right\}, \tag{33}$$

$$F_2 = +2\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1} \times \frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)} \times \operatorname{Argsinh}\left\{\frac{1}{2}\right\}, \tag{34}$$

$$F_3 = -2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{a}{A}+1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{a}{A}-1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}-1\right)^{-1}\right\}\right]\text{ and} \tag{35}$$

$$F_4 = -\left(\frac{a}{b}\right)\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{b}{B}+1\right)\times\text{Argsinh}\left\{\left(\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{b}{B}-1\right)\times\text{Argsinh}\left\{\left(\frac{b}{B}-1\right)^{-1}\right\}\right]. \tag{36}$$

The derived formulas hold for real case. If it is an ideal case then will hold that A→B→0 what presents that the insulating surface in the centre of annulus is very small. In such case you can imply following conditions:

$\left(\frac{a}{A}\pm1\right)\rightarrow\left(\frac{a}{A}\right)$ and $\left(\frac{b}{B}\pm1\right)\rightarrow\left(\frac{b}{B}\right)$

$$F_1 = +4\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\text{Argsinh}\left\{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right\}, \tag{37}$$

$$F_2 = +2\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\text{Argsinh}\left\{\frac{1}{2}\right\}, \tag{38}$$

$$F_3 = -2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{a}{A}\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}\right)^{-1}\right\}-\left(\frac{a}{A}\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}\right)^{-1}\right\}\right]=0\text{ and} \tag{39}$$

$$F_4 = -\left(\frac{a}{b}\right)\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left[\left(\frac{b}{B}\right)\times\operatorname{Argsinh}\left\{\left(\frac{b}{B}\right)^{-1}\right\}-\left(\frac{b}{B}\right)\times\operatorname{Argsinh}\left\{\left(\frac{b}{B}\right)^{-1}\right\}\right]=0. \qquad (40)$$

It is clear this is the optimal case when the sum of all functions will not be zero. It holds ($F_1 + F_2 + F_3 + F_4$) = ($F_1 + F_2$) $\neq 0$. It presents again the fact that the current annulus must be as narrow as possible, whereas, the potential annulus should be as wide as possible; it is that optimal variance. It comes in the only case when $k \neq \infty$ and that is why that $k^{-1} \neq 0$; therefore the voltage on the wide potential electrode is non-zero.

6 Analysis after the shape of electrode array determined by ratios (a/b) and (A/B)

Next theorizations can be done from the point of view how looks ratios (a/b) and (A/B). We distinguish two all different extremal cases. The first is when it is a circular electrode array, the second holds for a stretched elliptical electrode array. The second one tends to the electrode pad of Proximity Log.

For a = b you have (a/b) =1what presents circular annulus, whereas, for a ‹‹ b is (a/b) →0 and it is much stretched elliptical annulus. The same holds for characteristics A and B. If A=B you have (A/B) =1what presents circular annulus, whereas, for A‹‹B is (A/B) →0.

Formulas (37) and (38) present favourable case when the potential electrode is wide annulus with very small surface of the elliptical insulator in the centre. The current electrode is narrow annulus surrounding the potential one. Between both electrodes is then narrow surface of the elliptical insulator.

$$F_1 = +4\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\operatorname{Argsinh}\left\{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right\}, \qquad F_2 = +2\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\operatorname{Argsinh}\left\{\frac{1}{2}\right\}.$$

Circular electrode array

If holds that a=b and A=B you can imply the next conditions: (a/b) = 1 and (A/B) = 1.

$$F_1 = +2\times\left(\frac{a}{A}\right)^{-1}\times\operatorname{Argsinh}\left\{\frac{1}{2}\right\}, \qquad F_2 = +2\times\left(\frac{a}{A}\right)^{-1}\times\operatorname{Argsinh}\left\{\frac{1}{2}\right\} \text{ and } \qquad F_1 + F_2 = +4\times\left(\frac{a}{A}\right)^{-1}\times\operatorname{Argsinh}\left\{\frac{1}{2}\right\}.$$

The sum of functions F_1 and F_2 is nonzero.

Stretched elliptical electrode array (an elliptical electrode system with high eccentricity)

If holds that a ‹‹ b and A‹‹B then you can use conditions: (a/b) → 0 and (A/B) → 0.

$$F_1 = \lim_{(a/b)\to 0,(A/B)\to 0} +4\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times \operatorname{Argsinh}\left\{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right\}=$$

$$\lim_{(A/B)\to 0} +2\times\frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\lim_{(a/b)\to 0}\frac{\operatorname{Argsinh}\left\{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right\}}{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}}.$$

Through adjustment you have now multiplication of two limits. Partial limits you can solve separately.

$$\lim_{(A/B)\to 0} +2\times\frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)} = \lim_{(A/B)\to 0} +2\times\frac{1}{\left(\frac{b}{B}\right)+\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)} = +2\times\left(\frac{b}{B}\right)^{-1}.$$

$$\lim_{(a/b)\to 0}\frac{\operatorname{Argsinh}\left\{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right\}}{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}} = \lim_{(a/b)\to 0}\frac{\left\{\left[\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}\right]^2+1\right\}^{-\frac{1}{2}}\times\frac{1}{2}\times\left(\frac{a}{b}\right)^{-2}}{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-2}} = \lim_{(a/b)\to 0}\frac{1}{\frac{1}{2}\times\left(\frac{a}{b}\right)^{-1}} = \lim_{(a/b)\to 0} 2\times\left(\frac{a}{b}\right) = 0.$$

It holds that $F_1=0$. For F_2 you can write:

$$F_2 = \lim_{(A/B)\to 0} +2\times\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times \operatorname{Argsinh}\left\{\frac{1}{2}\right\} = \lim_{(A/B)\to 0} +2\times\frac{\left(\frac{A}{B}\right)}{\left(\frac{b}{B}\right)+\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)}\times \operatorname{Argsinh}\left\{\frac{1}{2}\right\} = 0.$$

It is variance for **the infinitely-stretched electrode array** when holds condition that (a/b) = 0 and (A/B) = 0. We get that $F_1 = F_2 = 0$. So the sum of both functions is zero too. It presents that infinitely- stretched electrode array is not convenient, however in practice such case cannot be.

For **the real stretched elliptical electrode array** being characterized with final dimensions when is (a/b) → 0 and (A/B) → 0, holds rather these formulas:

$$F_1 = +4\times\left(\frac{a}{b}\right)\times\left(\frac{b}{B}\right)^{-1},\ F_2 = +2\times \mathrm{Argsinh}\left\{\frac{1}{2}\right\}\times\frac{\left(\frac{A}{B}\right)}{\left(\frac{b}{B}\right)+\left(\frac{A}{B}\right)\times\left(\frac{a}{A}\right)} \approx +2\times \mathrm{Argsinh}\left\{\frac{1}{2}\right\}\times\left(\frac{A}{B}\right)\times\left(\frac{b}{B}\right)^{-1}.$$ It holds for a ‹‹ b and A‹‹B.

The stretched elliptical electrode array can be replaced with the rectangular electrodes which are easier for manufacturing.

7 Conclusions

On base of the made analysis there result these conclusions:

- It is possible to deduce all formulas for the partial constants k_{EM}, k_{EN}, k_{BM} and k_{BN} of elliptical electrodes. Due to them you can quickly and exactly count the mentioned constants in the numeric form. It is because you have data about the electrode characteristics that well-measurable are.
- It is possible to get the final formula for the circular electrodes too, thanks to implication of condition equality for both half-axes ellipses.
- It holds that the surface of the potential electrode annulus being inside of the electrode array should be as big as possible, whereas, the surface of annulus of the current annulus must be as narrow as possible. It is an optimal electrode array. The current electrode can look then like an elliptic/circular contour.

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