INFLUENCE OF SHAPE, DIMENSIONS AND DISTANCE OF ELECTRODES FOR CALCULATION OF THE MICRO-NORMAL CONSTANT OF MICROLOG

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Abstract: *This paper is about form of electrodes. There are the discoid electrodes generally used, however, the other form should not be problem. It remains a question of whether the square electrodes are as convenient as the disc ones are.*
Application of the square electrodes is unusual; however, studying various shapes than disc we can enlarge knowledge about Microlog. The square electrodes exist in two possible positions - to be situated on one of sides and on one of vertexes. The last named position remembers the card symbol remarked as diamond. Therefore you can remark such electrodes like the diamond electrodes.
The only angular rotation from side to vertex means fundamental change of the electric field around electrodes. Consequence of that are various courses of relations presented by two significant factors. There in near distances between electrodes acts strong influence of the magnification factor remarked as A/a. It is evident mainly for the square electrodes.
The diamond electrodes have this influence strong as well; exception is the case when A/a = 1. For both systems it holds that for long distances between electrodes the influence of the magnification factor decreases and you can observe only the influence of the translation factor remarked as 2m/a.
For enough long distances there are linearized curvilinear relationships and that is why you are allowed to use for enumeration of constant that equation presenting the point electrodes. The error in such case will be negligible.

Introduction

In the before paper there were derived formulas determined for counting of the micro-normal constant if the electrodes of Microlog are discs. I am sure here is offered the question of what shape of electric field is around the electrodes being of other shape geometry than disc. The easiest of all are the square electrodes. However, they have two different positions. They can be situated on one of four sides of square or on one of four vertexes. Such square looks like diamond; it is really a special diamond.
This paper investigates relation (k/a) = f (2m/a, A/a), when the first variable presents factor of translation and the second one factor of magnification. The electric fields of both positions of the square electrodes are various; different there is also the electric field of discoid electrodes. It holds, however, for short distances between electrodes only; for long distances all electric field incline to the field of the point electrodes.

Simulation of physical problem for the square electrodes

The current electrode has its side remarked as A; the potential electrode as a. It is Cartesian systems of coordinates: for variables (x, y) and for (k, h). It is depicted on fig.1.
The origin point of system remarked as (x, y) is located in an arbitrary point being on the surface of current electrode. The boundaries of integration for variable x are following:

$$x_2 = \left(\frac{a}{2}\right) \times \left(\frac{2m}{a} + \frac{2k}{a} + 1\right), \text{ and} \tag{1}$$

$$x_1 = \left(\frac{a}{2}\right) \times \left(\frac{2m}{a} + \frac{2k}{a} - 1\right). \tag{2}$$

The boundaries of integration for variable y are these:

$$y_2 = \left(\frac{a}{2}\right) \times \left(\frac{2n}{a} + \frac{2h}{a} + 1\right), \text{ and} \tag{3}$$

$$y_1 = \left(\frac{a}{2}\right) \times \left(\frac{2n}{a} + \frac{2h}{a} - 1\right). \quad (4)$$

Now, you need to define potential remarked like U_0 formed on the surface of potential electrode with the point current source in arbitrary point of the current electrode.

$$dU_0 = \frac{1}{4\pi} \times \left(\frac{R \times I}{\rho}\right) \times \frac{dS}{S}, \quad (5)$$

where ρ = the distance between the point current source being on surface of the current electrode and the point voltage element dU_0 being on surface of the potential electrode [m],
R = the resistivity of environment [Ωm], and
I = the current flowing through the current electrode [mA].

The surface of potential electrode it is surface of square.

$$S = a^2. \quad (6)$$

I can express potential U_0.

$$U_0 = \frac{1}{4\pi} \times \left(\frac{R \times I}{S}\right) \oiint_S \frac{dS}{\rho}. \quad (7)$$

This relation can be adjusted with the help of equation (6) on the form:

$$U_0 = \frac{1}{4\pi} \times \left(\frac{R \times I}{a}\right) \times a^{-1} \oiint_S \frac{1}{\sqrt{x^2 + y^2}}\, dy\, dx. \quad (8)$$

Equation (8) can be further adjusted.

$$U_0 = \frac{1}{4\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{2} \times \left(\frac{a}{2}\right)^{-1} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{1}{\sqrt{x^2 + y^2}}\, dy\, dx. \quad (9)$$

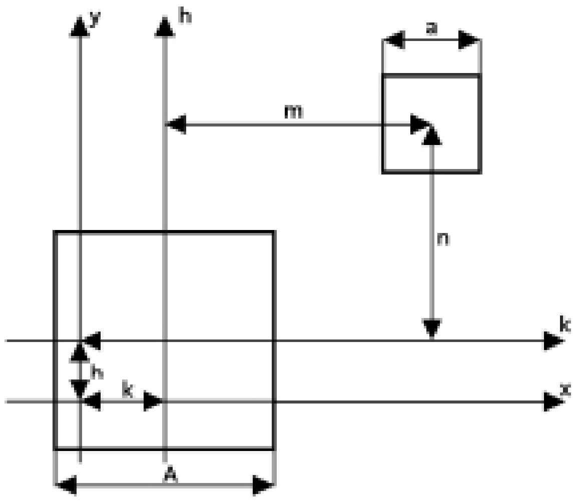

Fig.1: Basic schema of two Cartesian systems of coordinates for squared electrodes

You have to summarize an influence of all current sources forming the surface of the current electrode. The voltage element remarked as dU is following:

$$dU = U_0 \times \frac{dS}{S}, \text{ and} \quad (10)$$

$$S = A^2. \quad (11)$$

You need to integrate in the second system of coordinates (k, h). The origin point of system is laid in the centre of the current electrode as it is in fig.1. Boundaries of integration for coordinate k are these:

$$k_2 = +\frac{A}{2}\text{, and} \tag{12}$$

$$k_1 = -\frac{A}{2}\ . \tag{13}$$

The boundaries of integration for coordinate h are as follows:

$$h_2 = +\frac{A}{2}\text{, and} \tag{14}$$

$$h_1 = -\frac{A}{2}\ . \tag{15}$$

You can express voltage U with the help of relation (11):

$$U = \frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_1}^{k_2}\int_{h_1}^{h_2}\int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{1}{\sqrt{x^2 + y^2}}\, dy\, dx\, dh\, dk\ . \tag{16}$$

For the constant of the micro-normal it holds:

$$\frac{U}{R \times I} = \frac{1}{k} = \frac{F}{2\pi \times a}\ . \tag{17}$$

This equation results in relation:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{F}\ . \tag{18}$$

$$F = +\frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_1}^{k_2}\int_{h_1}^{h_2}\int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{1}{\sqrt{x^2 + y^2}}\, dy\, dx\, dh\, dk\ . \tag{19}$$

Equation (19) will be solved up owing to Laplace transformation.

$$\frac{1}{\sqrt{x^2 + y^2}} = \int_0^\infty J_0(w \times x) \times e^{-w \times y}\, dw\ . \tag{20}$$

As result of the step-by-step integration there are these formulas:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{\sum_{i=1}^{4} G_i}\ , \tag{21}$$

$$G_1 = +\frac{1}{4} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \left[\frac{2n}{a} + \left(\frac{A}{a} + 1\right)\right] \times \text{Argsinh}\left\{\frac{\frac{2n}{a} + \left(\frac{A}{a} + 1\right)}{\frac{2m}{a} + \left(\frac{A}{a} + 1\right)}\right\} - \left[\frac{2n}{a} + \left(\frac{A}{a} - 1\right)\right] \times \text{Argsinh}\left\{\frac{\frac{2n}{a} + \left(\frac{A}{a} - 1\right)}{\frac{2m}{a} + \left(\frac{A}{a} + 1\right)}\right\} \right\}, \tag{22}$$

$$G_2 = +\frac{1}{4} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \left[\frac{2n}{a} - \left(\frac{A}{a} + 1\right)\right] \times \text{Argsinh}\left\{\frac{\frac{2n}{a} - \left(\frac{A}{a} + 1\right)}{\frac{2m}{a} + \left(\frac{A}{a} + 1\right)}\right\} - \left[\frac{2n}{a} - \left(\frac{A}{a} - 1\right)\right] \times \text{Argsinh}\left\{\frac{\frac{2n}{a} - \left(\frac{A}{a} - 1\right)}{\frac{2m}{a} + \left(\frac{A}{a} + 1\right)}\right\} \right\}, \tag{23}$$

$$G_3 = -\frac{1}{4} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \sqrt{\left[\frac{2m}{a} + \left(\frac{A}{a} + 1\right)\right]^2 + \left[\frac{2n}{a} + \left(\frac{A}{a} + 1\right)\right]^2} - \sqrt{\left[\frac{2m}{a} + \left(\frac{A}{a} + 1\right)\right]^2 + \left[\frac{2n}{a} + \left(\frac{A}{a} - 1\right)\right]^2} \right\}\text{, and} \tag{24}$$

$$G_4 = -\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left[\frac{2n}{a}-\left(\frac{A}{a}+1\right)\right]^2}-\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left[\frac{2n}{a}-\left(\frac{A}{a}-1\right)\right]^2}\right\}. \quad (25)$$

Formulas could be derived thanks to integrals of complex variable and the inverse Laplace transformation. These formulas are adjusted in so way the fundament to be the potential electrode. This is fix, whereas, the current electrode to be supposed to be moveable. You receive function k/a = f (A/a, 2m/a, 2n/a). Note, please, the radius of the potential one is in denominator of all fractions. The mentioned ratios are defined like this:

A/a = the factor of magnification of the current electrode,
2m/a = the factor of translation of the current electrode in horizontal direction and
2n/a = the factor of translation of the current electrode in vertical direction.

Formulas from (21) up to (25) should be analyzed. If you have condition that n = 0 then it holds:

$$\left(\frac{2n}{a}\right) = 0. \quad (26)$$

In such case you attain these relations:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{(F_1+F_2)}, \quad (27)$$

$$F_1 = +\frac{1}{2}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left(\frac{A}{a}+1\right)\times \operatorname{Argsinh}\left\{\frac{\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}\right\}-\left(\frac{A}{a}-1\right)\times \operatorname{Argsinh}\left\{\frac{\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\left(\frac{A}{a}+1\right)}\right\}\Bigg\}, \text{ and} \quad (28)$$

$$F_2 = -\frac{1}{2}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left(\frac{A}{a}+1\right)^2}-\sqrt{\left[\frac{2m}{a}+\left(\frac{A}{a}+1\right)\right]^2+\left(\frac{A}{a}-1\right)^2}\right\}. \quad (29)$$

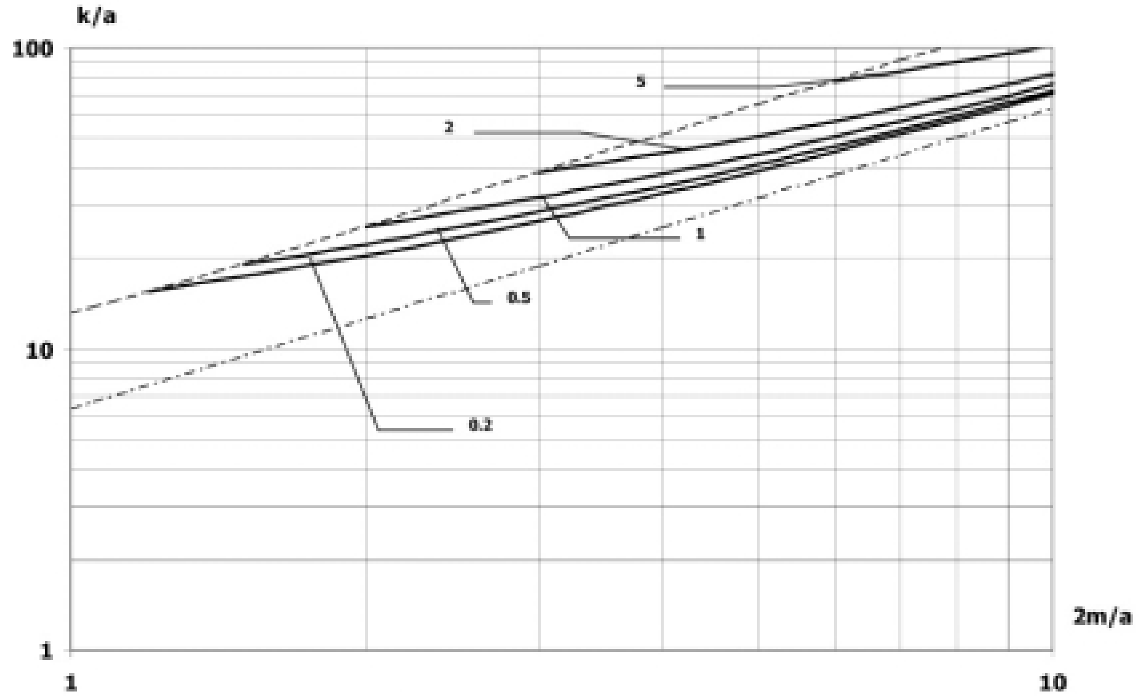

Fig.2: Relation k/a = f(2m/a, A/a)

It must be realized this inequality:

$$m \rangle \frac{a}{2}+\frac{A}{2}. \quad (30)$$

The above inequality can be adjusted like:

$$\left(\frac{2m}{a}\right) \rangle \left(\frac{A}{a}+1\right). \tag{31}$$

A/a	2m/a	k/a
0.0001	1.0001	13.058
0.001	1.001	13.069
0.050	1.050	13.666
0.100	1.100	14.278
0.200	1.200	15.511
0.300	1.300	16.754
0.400	1.400	18.006
0.500	1.500	19.265
1.000	2.000	25.631
2.000	3.000	38.530
3.000	4.000	51.512
4.000	5.000	64.525
5.000	6.000	77.554
6.000	7.000	90.591
7.000	8.000	103.633
8.000	9.000	116.678
9.000	10.000	129.726
10.000	11.000	142.776

Tab.1: Data for the envelope curve presenting the square electrodes

For A = a there is the following condition:

$$\frac{A}{a}=1. \tag{32}$$

The expressions being for F_1 and F_2 will get simpler.

$$F_1 = \text{Argsinh}\left\{\frac{2}{\frac{2m}{a}+2}\right\}, \text{ and} \tag{33}$$

$$F_2 = -\frac{1}{2}\times\left\{\sqrt{\left(\frac{2m}{a}+2\right)^2+4} - \left(\frac{2m}{a}+2\right)\right\}. \tag{34}$$

If you implement condition that (2m/a) >> 1, it will get valid:

$$\sqrt{\left(\frac{2m}{a}+2\right)^2+4} = \left(\frac{2m}{a}+2\right) = \left(\frac{2m}{a}\right). \tag{35}$$

Then you receive the expressions for F_1 and F_2.

$$F_1 = \text{Argsinh}\left\{2\times\left(\frac{2m}{a}\right)^{-1}\right\} = \ln\left\{2\times\left(\frac{2m}{a}\right)^{-1} + \sqrt{4\times\left(\frac{2m}{a}\right)^{-2}+1}\right\} = \ln\left\{2\times\left(\frac{2m}{a}\right)^{-1}+1\right\}, \text{ and} \tag{36}$$

$$F_2 = 0. \tag{37}$$

If you return again to condition (2m/a) >> 1, you will be allowed to implement the next easier relation:

$$\ln\left\{ 2\times\left(\frac{2m}{a}\right)^{-1} + 1 \right\} = 2\times\left(\frac{2m}{a}\right)^{-1} = \left(\frac{2m}{a}\right)^{-1}. \tag{38}$$

Thus, in this way you have that:

$$F_1 = \left(\frac{2m}{a}\right)^{-1} \text{, and} \tag{39}$$

$$F_2 = 0 .$$

By substitution equations (37) and (39) it is possible to attain the following equation:

$$\left(\frac{k}{a}\right) = 2\pi\times\left(\frac{2m}{a}\right). \tag{40}$$

2m/a	k/a
1	6.283
2	12.566
3	18.850
4	25.133
5	31.416
6	37.699
7	43.982
8	50.265
9	56.549
10	62.832

Tab.2: Data for linear relation presenting the point electrodes

You can easy make sure that it is the equation for the point electrodes. It presents asymptote for function k/a = f (A/a, 2m/a) which is depicted on fig.2. Trend to equation (40) for condition (2m/a) >> 1 is distinctly undoubted. Relations for ratios A/a are more influenced by dimensions of electrodes then it was for the disc electrodes.

Simulation of the physical problem for the diamond electrodes

This case is presented like system of the diamond electrodes. There are two Cartesian subsystems again. The first having coordinates x and y and the second with coordinates k and h. It is depicted in fig.3.

The origin point for x and y is situated again in arbitrary point of the current electrode. However, the boundaries of integration have more complicated expressions than it is for the before event.

There exist these boundaries of coordinate x:

$$x = \langle\, k + m \,,\, k + m + a\times\frac{\sqrt{2}}{2} \,\rangle \text{, and} \tag{41}$$

$$x = \langle\, k + m - a\times\frac{\sqrt{2}}{2} \,,\, k + m \,\rangle . \tag{42}$$

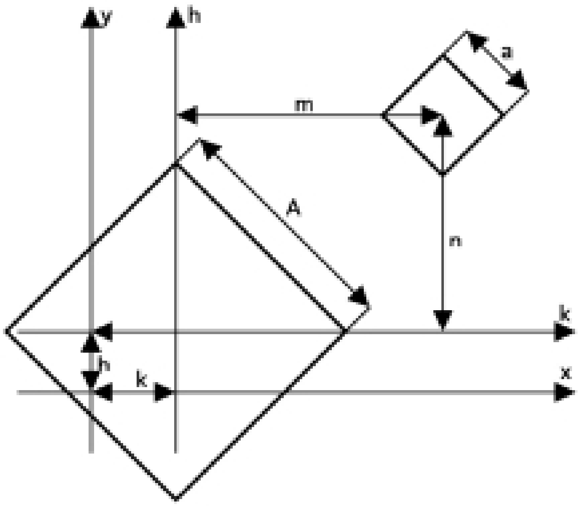

Fig.3: Basic schema of two Cartesian systems of coordinates for diamond electrodes

This is given by the need of integration for two triangles of the potential electrode. In such case you have the boundaries of integration for each of ones.

For the first triangle you get the following boundaries:

$$x_{12} = \left(\frac{a}{2}\right) \times \left(\frac{2m}{a} + \frac{2k}{a} + \sqrt{2}\right), \text{ and} \quad (43)$$

$$x_{11} = \left(\frac{a}{2}\right) \times \left(\frac{2m}{a} + \frac{2k}{a}\right). \quad (44)$$

For the second triangle you receive these boundaries:

$$x_{22} = \left(\frac{a}{2}\right) \times \left(\frac{2m}{a} + \frac{2k}{a}\right), \text{ and} \quad (45)$$

$$x_{21} = \left(\frac{a}{2}\right) \times \left(\frac{2m}{a} + \frac{2k}{a} - \sqrt{2}\right). \quad (46)$$

Contours of the potential electrode are determined with these lines:

$$y = -x + (m+n) + (k+h) - a \times \frac{\sqrt{2}}{2}, \quad (47)$$

$$y = -x + (m+n) + (k+h) + a \times \frac{\sqrt{2}}{2}, \quad (48)$$

$$y = +x - (m-n) - (k-h) + a \times \frac{\sqrt{2}}{2}, \text{ and} \quad (49)$$

$$y = +x - (m-n) - (k-h) - a \times \frac{\sqrt{2}}{2}. \quad (50)$$

Boundaries of integration for the first triangle are as follows:

$$y_{12} = \left(\frac{a}{2}\right) \times \left(-\frac{2x}{a} + \frac{2m}{a} + \frac{2n}{a} + \frac{2k}{a} + \frac{2h}{a} + \sqrt{2}\right), \text{ and} \quad (51)$$

$$y_{11} = \left(\frac{a}{2}\right) \times \left(+\frac{2x}{a} - \frac{2m}{a} + \frac{2n}{a} - \frac{2k}{a} + \frac{2h}{a} - \sqrt{2}\right). \quad (52)$$

Boundaries of integration for the second triangle are these:

$$y_{22} = \left(\frac{a}{2}\right) \times \left(+\frac{2x}{a} - \frac{2m}{a} + \frac{2n}{a} - \frac{2k}{a} + \frac{2h}{a} + \sqrt{2} \right), \text{ and} \quad (53)$$

$$y_{21} = \left(\frac{a}{2}\right) \times \left(-\frac{2x}{a} + \frac{2m}{a} + \frac{2n}{a} + \frac{2k}{a} + \frac{2h}{a} - \sqrt{2} \right). \quad (54)$$

The potential remarked as U_0 generated with the only point of the current source being in an arbitrary point of surface of the current electrode is defined with this expression:

$$U_0 = \frac{1}{4\pi} \times \left(\frac{R \times I}{\rho}\right) \oint\!\!\int_S \frac{dS}{\rho}, \text{ and}$$

$$S = a^2 .$$

After substitution for S and for boundaries of integration you will obtain expression for U_0:

$$U_0 = +\frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{4} \times \left(\frac{a}{2}\right)^{-1} \int_{x_{11}}^{x_{12}} \int_{y_{11}}^{y_{12}} \frac{1}{\sqrt{x^2 + y^2}}\, dx\, dy$$

$$+\frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{4} \times \left(\frac{a}{2}\right)^{-1} \int_{x_{21}}^{x_{22}} \int_{y_{21}}^{y_{22}} \frac{1}{\sqrt{x^2 + y^2}}\, dx\, dy . \quad (55)$$

Effect of all point current sources forming surface of the current electrode is determined with their sum. The voltage element remarked as dU is defined like this:

$$dU = U_0 \times \frac{dS}{S}, \text{ and}$$

$$S = A^2 .$$

The origin point of Cartesian system of coordinates k and h is in the centre of the current electrode again. Integration is realized for two triangles. Boundaries of integration for coordinate k are following:

$$k_{12} = +\left(\frac{A}{2}\right) \times \sqrt{2} , \quad (56)$$

$$k_{11} = 0 , \quad (57)$$

$$k_{22} = 0 , \text{ and} \quad (58)$$

$$k_{21} = -\left(\frac{A}{2}\right) \times \sqrt{2} . \quad (59)$$

Contours of the current electrode are defined by these lines:

$$h = +k + A \times \frac{\sqrt{2}}{2} , \quad (60)$$

$$h = -k - A \times \frac{\sqrt{2}}{2} , \quad (61)$$

$$h = -k + A \times \frac{\sqrt{2}}{2} , \text{ and} \quad (62)$$

$$h = +k - A \times \frac{\sqrt{2}}{2} . \quad (63)$$

The above lines determine the boundaries of integration for coordinate h.

$$h_{12} = \left(\frac{A}{2}\right) \times \left(-\frac{2k}{A} + \sqrt{2} \right), \quad (64)$$

$$h_{11} = \left(\frac{A}{2}\right) \times \left(+\frac{2k}{A} - \sqrt{2} \right), \quad (65)$$

$$h_{22} = \left(\frac{A}{2}\right) \times \left(+ \frac{2k}{A} + \sqrt{2} \right), \text{ and} \tag{66}$$

$$h_{21} = \left(\frac{A}{2}\right) \times \left(- \frac{2k}{A} - \sqrt{2} \right). \tag{67}$$

Now, all is ready for expression of potential remarked as U like sum of four integrals.

$$\begin{aligned} U = &+ \frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{11}}^{k_{12}} \int_{h_{11}}^{h_{12}} \int_{x_{11}}^{x_{12}} \int_{y_{11}}^{y_{12}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \\ &+ \frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{11}}^{k_{12}} \int_{h_{11}}^{h_{12}} \int_{x_{21}}^{x_{22}} \int_{y_{21}}^{y_{22}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \\ &+ \frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{21}}^{k_{22}} \int_{h_{21}}^{h_{22}} \int_{x_{11}}^{x_{12}} \int_{y_{11}}^{y_{12}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \\ &+ \frac{1}{2\pi} \times \left(\frac{R \times I}{a}\right) \times \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{21}}^{k_{22}} \int_{h_{21}}^{h_{22}} \int_{x_{21}}^{x_{22}} \int_{y_{21}}^{y_{22}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \; . \end{aligned} \tag{68}$$

For the constant it will be valid formula (18):

$$\left(\frac{k}{a}\right) = \frac{2\pi}{F}, \text{ and}$$

$$\begin{aligned} F = &+ \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{11}}^{k_{12}} \int_{h_{11}}^{h_{12}} \int_{x_{11}}^{x_{12}} \int_{y_{11}}^{y_{12}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \\ &+ \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{11}}^{k_{12}} \int_{h_{11}}^{h_{12}} \int_{x_{21}}^{x_{22}} \int_{y_{21}}^{y_{22}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \\ &+ \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{21}}^{k_{22}} \int_{h_{21}}^{h_{22}} \int_{x_{11}}^{x_{12}} \int_{y_{11}}^{y_{12}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \\ &+ \frac{1}{16} \times \left(\frac{a}{2}\right)^{-1} \times \left(\frac{A}{2}\right)^{-2} \int_{k_{21}}^{k_{22}} \int_{h_{21}}^{h_{22}} \int_{x_{21}}^{x_{22}} \int_{y_{21}}^{y_{22}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx \, dh \, dk \; . \end{aligned} \tag{69}$$

Formula (69) is solved with the help of relation (20) again. The following formulas are presented like final result of integration.

$$\left(\frac{k}{a}\right) = \frac{2\pi}{\sum_{i=1}^{4} G_i},$$

$$G_1 = + \frac{1}{8} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \left[\frac{2n}{a} + \sqrt{2} \times \left(\frac{A}{a} + 1 \right) \right] \times \text{Argsinh} \left\{ \frac{\frac{2n}{a} + \sqrt{2} \times \left(\frac{A}{a} + 1 \right)}{\frac{2m}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1 \right)} \right\} - \left[\frac{2n}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1 \right) \right] \times \text{Argsinh} \left\{ \frac{\frac{2n}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1 \right)}{\frac{2m}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1 \right)} \right\} \right\}, \tag{70}$$

$$G_2 = + \frac{1}{8} \times \left(\frac{A}{a}\right)^{-1} \times \left\{ \left[\frac{2n}{a} - \sqrt{2} \times \left(\frac{A}{a} + 1 \right) \right] \times \text{Argsinh} \left\{ \frac{\frac{2n}{a} - \sqrt{2} \times \left(\frac{A}{a} + 1 \right)}{\frac{2m}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1 \right)} \right\} - \left[\frac{2n}{a} - \sqrt{2} \times \left(\frac{A}{a} - 1 \right) \right] \times \text{Argsinh} \left\{ \frac{\frac{2n}{a} - \sqrt{2} \times \left(\frac{A}{a} - 1 \right)}{\frac{2m}{a} + \sqrt{2} \times \left(\frac{A}{a} - 1 \right)} \right\} \right\}, \tag{71}$$

$$G_3 = -\frac{1}{8}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]^2} - \sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2}\right\}, \text{ and} \tag{72}$$

$$G_4 = -\frac{1}{8}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]^2} - \sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+\left[\frac{2n}{a}-\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2}\right\}. \tag{73}$$

If you introduce again condition (26) that (2n/a) = 0, you will receive these formulas:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{(F_1 + F_2)},$$

$$F_1 = +\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\left[\sqrt{2}\times\left(\frac{A}{a}+1\right)\right]\times \text{Argsinh}\left\{\frac{\sqrt{2}\times\left(\frac{A}{a}+1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\} - \left[\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]\times \text{Argsinh}\left\{\frac{\sqrt{2}\times\left(\frac{A}{a}-1\right)}{\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)}\right\}\right\}, \text{ and} \tag{74}$$

$$F_2 = -\frac{1}{4}\times\left(\frac{A}{a}\right)^{-1}\times\left\{\sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+2\times\left(\frac{A}{a}+1\right)^2} - \sqrt{\left[\frac{2m}{a}+\sqrt{2}\times\left(\frac{A}{a}-1\right)\right]^2+2\times\left(\frac{A}{a}-1\right)^2}\right\}. \tag{75}$$

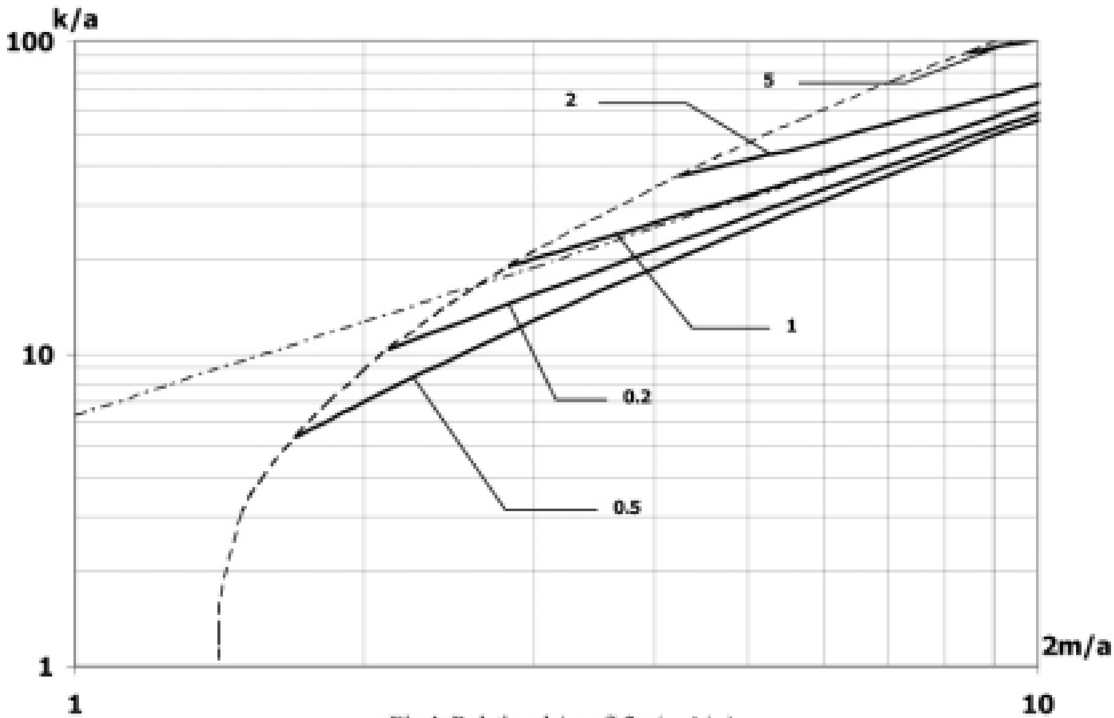

Fig.4: Relation k/a = f(2m/a, A/a)

Further, this must be valid:

$$m > \frac{\sqrt{2}}{2}\times(a + A). \tag{76}$$

Inequality can be written down like this:

$$\left(\frac{2m}{a}\right) > \sqrt{2}\times\left(1+\frac{A}{a}\right). \tag{77}$$

If you implement condition (32) that (A/a) = 1, you will attain the following expressions:

$$F_1 = \frac{\sqrt{2}}{2} \times \text{Argsinh}\left\{ 2\sqrt{2} \times \left(\frac{2m}{a}\right)^{-1} \right\} = \frac{\sqrt{2}}{2} \times \ln\left\{ 2\sqrt{2} \times \left(\frac{2m}{a}\right)^{-1} + \sqrt{\left[2\sqrt{2} \times \left(\frac{2m}{a}\right)^{-1}\right]^2 + 1} \right\}, \text{a} \quad (78)$$

$$F_2 = -\frac{1}{4} \times \left\{ \sqrt{\left(\frac{2m}{a}\right)^2 + 8} - \left(\frac{2m}{a}\right) \right\}. \quad (79)$$

A/a	2m/a	k/a
0.0002	1.4145	1.044
0.0005	1.4149	1.167
0.0008	1.4153	1.243
0.001	1.4156	1.285
0.005	1.4213	1.677
0.050	1.4850	2.965
0.100	1.556	3.847
0.200	1.697	5.412
0.300	1.838	6.971
0.400	1.980	8.587
0.500	2.121	10.249
1.000	2.828	19.018
2.000	4.243	37.217
3.000	5.657	55.587
4.000	7.071	74.004
5.000	8.485	92.440
6.000	9.899	110.885
7.000	11.314	129.341
8.000	12.728	147.795
9.000	14.142	166.251
10.000	15.556	184.709

Tab.3: Data for the envelope curve presenting the diamond electrodes

For (2m/a) >> 1 it holds that:

$$\sqrt{\left(\frac{2m}{a}\right)^2 + 8} = \left(\frac{2m}{a}\right). \quad (80)$$

For F_1 and F_2 you obtain these expressions:

$$F_1 = \frac{\sqrt{2}}{2} \times \ln\left\{ 2\sqrt{2} \times \left(\frac{2m}{a}\right)^{-1} + 1 \right\} = \frac{\sqrt{2}}{2} \times 2\sqrt{2} \times \left(\frac{2m}{a}\right)^{-1} = 2 \times \left(\frac{2m}{a}\right)^{-1} = \left(\frac{2m}{a}\right)^{-1}, \text{and} \quad (81)$$

$$F_2 = 0.$$

These values tend again to formula (40).

$$\left(\frac{k}{a}\right) = 2\pi \times \left(\frac{2m}{a}\right).$$

Function k/a = f (2m/a, A/a) in accordance to equations (74) and (75) and also to equation (27) is depicted in fig. 4. This function has also as asymptote the relationship for the point electrode remarked as (40).

Comparison of different electrode shapes

In comparison to the disc electrodes the square and diamond electrodes have their curves more curved than it is for the disc electrodes. About almost linear relationship we cannot speak for the square electrodes at all. The diamond electrodes have exception for ratio remarked that A/a = 1. This curve is almost linear and can be replaced by relation for the point electrodes.

For standard geometry of Microlog there hold these values of factors: (2m/a) = 5, (2m/a) = 10 and (A/a) = 1. Tab.4 presents comparison of almost linear relations for the diamond electrodes, the disc electrodes and for the point electrodes. For ratios (k/a) it holds that the lowest values are for the point electrodes, whereas, the highest ones are for the diamond electrodes; nevertheless, differences are so tiny that counting of constant k provides data being identical with precision on two decimal places.

Form of electrodes	A/a	2m/a	k/a	a[m]	k[m]	A/a	2m/a	k/a	a[m]	k[m]
The point electrodes		5	31.416	0.01	0.31		10	62.832	0.01	0.63
The disc electrodes	1	5	31.821	0.01	0.32	1	10	63.040	0.01	0.63
The diamond electrodes	1	5	32.202	0.01	0.32	1	10	63.244	0.01	0.63

Tab.4: Comparison three different form of electrodes having A/a = 1 for values of constant k

Fig.2 and fig. 4 present relations (k/a) = (2m/a, A/a).There are depicted also the envelope curve like dashed line and asymptote of relations after equation (40) like dash-and –dot line.

Conclusions

On the base of derived formulas and constructed plots I have got these conclusions:

1. The geometry shape of electrodes affects the electric field around and it has great significance when constant of the micro-normal is enumerated.
2. Electric field of the diamond electrodes in the only case when (A/a) = 1 can be classified like almost linear. All resting events present curvilinear relationships. It holds completely for all relations of the square electrodes.
3. If factor of translation is much bigger than one, the electric field will incline to field of the point electrodes. The result is linear relationship presenting asymptote for all relations depicted. It holds not only for the square and diamond electrodes, but, for the disc electrodes, as well.

References

Dachnow V. N. (1967): Elektritcheskye i magnitnye metody issledovania skvazhin, Fundament of theory, Nedra, - Moscow

Poznámka redakce:
Z důvodu množství vzorců byl článek na stránce uspořádán do jednoho textového sloupce.
Editorial Board remark:
The lot of long arithmetical formulas in the text has been the reason for different arrangement of this paper.