THE GEOLOGICAL AGE OF SEDIMENTS AFTER TYPE OF THE DIAGENESIS EVOLUTION

GEOLOGICKÉ STÁŘÍ SEDIMENTŮ PODLE TYPU DIAGENETICKÉHO VÝVOJE

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Abstract

This paper is dealing with determination of geological age of rocks. The age is enumerated after formulas derived from differential equation for the curves of evolution. Velocity of sedimentary consolidation is directed by laws being closed to one normal and two lognormal distributions with positive and negative asymmetry. Relationship being between the bygone time and total porosity is characterized by the characteristic curve of evolution having several stages: compaction, compression and cementation.

There are three different types of sedimentary evolution: the accelerated one, the normal one and the delayed one. Various types of evolution make possible explain those rocks of different geological age having the same porosity; however too, that case when the rock of identical age can go through with three various evolutions characterized by three different porosities one another.

The curve of total porosity is transformed to the curve of the apparent geological time. We have to find its minimal values; we insert through the curve of trend presenting the real geological time. Both curves are continuous; each depth point has its own time.

Abstrakt

Tato práce se zabývá určováním geologického stáří hornin. Stáří se vyhodnocuje podle vzorců odvozených z diferenciální rovnice křivek vývoje. Rychlost zpevňování sedimentů se řídí zákony, které jsou blízké normálnímu zákonu rozdělení a dvěma lognormálním zákonům rozdělení, které mají kladnou a zápornou asymetrii. Závislost, která existuje mezi uplynulým časem a celkovou pórovitostí sedimentů je charakterizována vybranou křivkou vývoje, která má několik fází: rychlé zpevnění, pomalé stlačování a cementaci pórů.

Existují tři rozdílné typy sedimentárního vývoje. Zrychlený, normální a zpomalený vývoj. Různý typ vývoje sedimentů umožňuje vysvětlit nejen to, proč horniny rozdílného geologického stáří mají identickou pórovitostí, ale také ten případ, kdy horniny stejného stáří mohou projít rozdílným vývojem charakterizovaným až třemi rozdílnými pórovitostmi.

Křivku celkové pórovitosti transformujeme na křivku zdánlivého geologického času. Nalezneme jeho minimální hodnoty a jimi proložíme křivku trendu, představující skutečný geologický čas. Obě křivky mají spojitý charakter, každému hloubkovému bodu tak odpovídá příslušný čas.

Keywords

sedimentary basins, curves of diagenesis evolution, accelerated, normal and delayed types of the process of compaction related to normal and/or lognormal distribution

1 Introduction

The diagenesis evolution is determined by types of the sedimentary consolidation. I suppose an existence of three basic ones there: the normal with normal distribution of porosity, the accelerated having lognormal distribution with positive asymmetry and the delayed characterized with lognormal distribution having negative asymmetry.

The normal one has normal distribution of statistical set of total porosity. It presents many various factors influencing the diagenesis evolution, however, no of them dominates over remaining ones. Note, please, yet the fact that the values of total porosity being very high and very low are not frequent. This supports premise the law of normal distribution is most frequent.

The accelerated one is characterized with lognormal distribution having positive asymmetry. One or more factors prevail over those resting and they accelerate process of sedimentary consolidation.

The delayed one has lognormal distribution with negative asymmetry. It is similar to previous one, but, its asymmetry is contradictory. One, two or more factors prevail again over the remaining ones; however, those several factors delay the process of sedimentary consolidation.

All three mentioned styles present fundament for derivation of the curves of evolution expressing process of sedimentary consolidation between the bygone geological time and total porosity of rocks. It holds that the ancient rocks have lower porosity and the recent rocks higher porosity. However, determination of geological age of sediments after total porosity has a lot of exceptions. For example; the sediments of Lower Cambrium in the area of Baltic Sea are not consolidated at all and in spite of that they look very recently, whereas, the Pleistocene sediments of Nagelfluh Formation in Alps are perfectly consolidated. KETTNER (1957), 2, p. 270. And this is only one of similar exceptions. Such observations are confirmed too from other parts of world by next authors. They are for example REINECK, H. E. and SINGH, I. B. (1975), SERRA, O. (1986), further, SERRA, O. and SULPICE, L. (1975), and finally, SERRA, O.and SERRA, L. (2003). The described fact is well-explainable, of course, thanks to three different types of sedimentary consolidation when an identical rock of the same age can have three different porosities and , inversely, the same porosity can present three various geological ages. Therefore, the right interpretation of geological age is unambiguously depended on the known type of sedimentary consolidation.

2 Hypothesis of the supposed problem

The brainwave to write this paper originated over one textbook of mathematics, REKTORYS et al. (1968), over the chapter the law of growth and logistic curves. It seemed me very interesting to put it in geology, to describe the diagenesis evolution of sediments and to determine an absolute geologic age of them.

In the oil geology geologists use so called the age borehole sections. They present different geological periods put together like formations in the depth. The time data are taken after findings of palaeontology, however, much more from micropalaeontology. This paper offers something similar; it goes out, however, from well-logging data and this is the next and independent source of information. So one can compare two age borehole sections; each of them from various principles. It offers their comparing, their correlation in the age and, too, to develop next investigation.

It is about the evolution curves. Solution of the differential equation of type dx/dt = f(x) is named like the law of evolution. REKTORYS et al. (1968). In this case it is dp/dt = f (p). The mentioned equation presents a change of total porosity with geological time thanks to function f (p) which has closed relation to function f^*(p) presenting one diagenesis evolution . That function can have one of three possible variances of sedimentary consolidation. Function f^*(p) is determined from well-logging data of total porosity versus depth of the each borehole. The function is characterized by the mean of statistical set of total porosity μ and by the standard deviation of the same statistical set σ. The numeric values of both constants correspond to the geological time which has gone from the moment when flowing quick sediment became solid dead one. That means that the mentioned constants are changed too in the geological time. The process of compaction is infinitely slow dynamic process going on towards future. And therefore each well-logging evaluation of total porosity presents a snap in the geological time having its own values for μ and σ. The new snap made some millions later can be completely other.

The differential equation dp/dt = f (p) makes possible enumerate the geological time t. As it was said, the time is taken from the moment when total porosity of sediment is lower than 50%; it correspond to the state when flowing quick sediment will become solid dead one.

3 Derivation of the evolution curve for normal sedimentary consolidation

I shall briefly touch functions f*(p) and f (p). Function f*(p) presents statistic set of well-logging data obtained from the borehole in the form of continuous curve with the depth, when each value of total porosity belongs to a certain borehole point. I suppose that the made statistic set carries information about diagenesis evolution of sediments in the investigated depth interval.

Processes influencing this evolution are very much; some of them are known, the remaining not. If no of processes does not dominate over others, it is normal distribution of the set. However, when one or more processes dominate, it is a lognormal distribution. Such set is deflected to the left/to the right and presents the accelerated /delayed evolutions. It is described in journal *Acta Universitatis Carolinae – Geologica 2004*, RYŠAVÝ (2004). The function presenting the statistic set with the help of two characteristic σ and μ characterizes diagenesis evolution and is remarked as f*(p).

Function f (p) being on the right side of the differential equation must be very close to the above function f*(p) and to use, too, characteristics σ and μ. However, it cannot be identical. Just this were reasons why I selected function f (p) as written in this paper, the function differing only in the sign for exponent from function f*(p).

The basic equation is the differential one. Such equation says that the velocity of sedimentary consolidation equals to the law being similar to the law of normal distribution and using its characteristics, however, in spite of that is a bit other. This equation supposes, as well, that between the bygone time and total porosity there exist an inverse relation. The higher porosity presents younger sediments, the lower porosity the older sediments.

The law of normal distribution has this form:

$$f^*(p)=\frac{1}{\sigma\sqrt{2\pi}}\times\exp\left\{-\frac{1}{2}\times\left(\frac{p-\mu}{\sigma}\right)^2\right\}, \tag{1}$$

where p = total porosity of rocks,

t = the apparent time gone by from the beginning of sedimentary consolidation,

μ = the mean of statistical set of total porosity, and

σ = the standard deviation of the same statistical set.

The law which will be used has this shape:

$$\frac{dp}{dt}=f(p)=\frac{1}{\sigma\sqrt{2\pi}}\times\exp\left\{+\frac{1}{2}\times\left(\frac{p-\mu}{\sigma}\right)^2\right\}. \tag{2}$$

The law has both characteristics μ and σ, because both carry correct information about total porosity in the borehole profile section. The law, but, supposes an intensive exponential changes of porosity.

Note, please, the dimension of time unit is not defined yet; it will be done for all three types of distribution in common in one of next chapters. Due to equation (2) it is possible to write that:

$$dt=\sigma\sqrt{2\pi}\times\exp\left\{-\frac{1}{2}\times\left(\frac{p-\mu}{\sigma}\right)^2\right\}dp\,. \tag{3}$$

Now, you can express time like that:

$$t=\sigma\sqrt{2\pi}\int_{p_{\max}}^{p}\exp\left\{-\frac{1}{2}\times\left(\frac{p-\mu}{\sigma}\right)^2\right\}dp\ \ldots\text{for } p_{\max}\rangle\, p\,. \tag{4}$$

If you use substitution q = [(p-μ)/σ], you will attain this expression:

$$t=\sigma^2\sqrt{2\pi}\int_{[(p_{\max}-\mu)/\sigma]}^{[(p-\mu)/\sigma]}\exp\left\{-\frac{1}{2}\times(q)^2\right\}dq\,. \tag{5}$$

Due to division of integral on two ones you obtain this formula:

$$t=-\pi\sigma^2\times\frac{2}{\sqrt{2\pi}}\int_{0}^{[(p_{\max}-\mu)/\sigma]}\exp\left\{-\frac{1}{2}q^2\right\}dq+\pi\sigma^2\times\frac{2}{\sqrt{2\pi}}\int_{0}^{[(p-\mu)/\sigma]}\exp\left\{-\frac{1}{2}q^2\right\}dq\,. \tag{6}$$

You will get both integrals like Laplace's Function.

$$t = (t-0) = -\pi\sigma^2 \times \left\{ \Phi\left(\frac{p_{\max}-\mu}{\sigma}\right) - \Phi\left(\frac{p-\mu}{\sigma}\right) \right\}. \quad (7)$$

where Φ (q) = Laplace's Function.

It is needed to say why the time is negative. Every time interval has two important points; the start point and the end point. The time runs from the zero time to the recent time. However, what is crucial is which of the above time points an observer is in. For formula (7) it is the end point presenting the recent time. That means the past and negative time values. However, formula (7) can be adjusted so the right side of equation to be positive.

$$-t = -(t-0) = +\pi\sigma^2 \times \left\{ \Phi\left(\frac{p_{\max}-\mu}{\sigma}\right) - \Phi\left(\frac{p-\mu}{\sigma}\right) \right\}. \quad (8)$$

Formula (8) is the second possible case. The time again runs from the zero time to the recent time, but, the observer is positioned in the start point what presents the future and positive values of geological time. Formulas (7) and (8) present in the absolute value the same fundamental relation expressing the function being between the bygone time and total porosity. It is not important where an observer is, but, the absolute value of the bygone time. Thus, it is possible to write:

$$|t| = \pi\sigma^2 \times \left\{ \Phi\left(\frac{p_{\max}-\mu}{\sigma}\right) - \Phi\left(\frac{p-\mu}{\sigma}\right) \right\}. \quad (9)$$

Let's express Laplace's Function like series.

$$\Phi(q) = \frac{2}{\sqrt{2\pi}} \times \int_0^q e^{-\frac{x^2}{2}}\, dx = \frac{2}{\sqrt{2\pi}} \times \int_0^q \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} \times \frac{x^{2k}}{k!}\, dx = \frac{2}{\sqrt{2\pi}} \times \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} \times \frac{1}{k!} \int_0^q x^{2k}\, dx =$$
$$= \frac{2}{\sqrt{2\pi}} \times \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} \times \frac{1}{k!} \times \frac{(q)^{2k+1}}{(2k+1)} = \frac{2}{\sqrt{2\pi}} \times \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} \times \frac{1}{k!} \times \frac{q^{(2k+1)}}{(2k+1)}. \quad (10)$$

Equation (9) can be adjusted then like this:

$$|t| = \pi\sigma^2 \times \frac{2}{\sqrt{2\pi}} \times \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} \times \frac{1}{k!} \times \frac{1}{(2k+1)} \times \left\{ \left(\frac{p_{\max}-\mu}{\sigma}\right)^{(2k+1)} - \left(\frac{p-\mu}{\sigma}\right)^{(2k+1)} \right\}. \quad (11)$$

For k = 0 you will take only the first term of series and the all remaining you will neglect. So you receive this approximate formula being very rough but for fundamental information very important:

$$|t| \approx \pi\sigma^2 \times \frac{2}{\sqrt{2\pi}} \times \left\{ \left(\frac{p_{\max}-\mu}{\sigma}\right) - \left(\frac{p-\mu}{\sigma}\right) \right\} = \sigma^2\sqrt{2\pi} \times \left(\frac{p_{\max}-p}{\sigma}\right). \quad (12)$$

The equation you can again adjust like that.

$$|\,t\,| \approx -\sigma\sqrt{2\pi}\times p + \sigma\sqrt{2\pi}\times p_{\max}. \qquad (13)$$

This is very important formula. It confirms that fundamental premise; the older sediment is the lower porosity has. The older sediments have low porosity, whereas, the younger sediments are characterized by high porosity. It holds for identical type of law, of course. You see, too, that for $p = p_{max} = 0.50$ there is $|\,t\,| = 0$ what confirms that 50% presents the crucial point from the geologic time run. Formula (13) is, really, very rough estimation and I do not use it. Best of all is to use formula (9) which has completely all higher terms.

Formula (9) is more precise than formula (13). It makes possible to review information about the single phases of consolidation. This is the basic formula making possible calculate the geological age of rocks when the law closed to the law of normal distribution was supposed. The difference between both Laplace's Functions can present a weighted contribution of all factors having been acting in the bygone time. The function $p = f(t)$ presents the curve of behaviour of porosity. It is depicted in fig.1.

As it holds that $p_{max} > p$, for values of Laplace's Function you will get the following inequalities:

$$+1 > \Phi\left(\frac{p_{\max}-\mu}{\sigma}\right) > \Phi\left(\frac{p-\mu}{\sigma}\right) > \Phi\left(-\frac{\mu}{\sigma}\right) > -1. \qquad (14)$$

Relation $-t = f(p)$ allows us to determine not only the time which had gone by yet, but also, the time remaining up to that time moment when porosity will be zero. We shall start from two formulas for $(-t)$ and $(-t_0)$.

$$-t = \pi\sigma^2\times\left\{\Phi\left(\frac{p_{\max}-\mu}{\sigma}\right)-\Phi\left(\frac{p-\mu}{\sigma}\right)\right\}.$$

$$-t_0 = \pi\sigma^2\times\left\{\Phi\left(\frac{p_{\max}-\mu}{\sigma}\right)-\Phi\left(-\frac{\mu}{\sigma}\right)\right\} = \pi\sigma^2\times\left\{\Phi\left(\frac{p_{\max}-\mu}{\sigma}\right)+\Phi\left(\frac{\mu}{\sigma}\right)\right\}. \qquad (15)$$

In formula (14) there was used relation for negative argument of Laplace's Function:

$$\Phi\left(-\frac{\mu}{\sigma}\right) = -\Phi\left(\frac{\mu}{\sigma}\right). \qquad (16)$$

Now, we are able to express the time remaining till the moment when all pore space will be filled with cement. Total porosity has then zero value, $p = 0$.

$$(-t)-(-t_0) = (t_0 - t) = -\pi\sigma^2\times\left\{\Phi\left(\frac{p-\mu}{\sigma}\right)+\Phi\left(\frac{\mu}{\sigma}\right)\right\}. \qquad (17)$$

This is the case when the observer is in the end point of the time interval presenting the time of completion of all consolidation process. From this point of view it is the past and values are negative. For interval $-(t_0 - t) = (t - t_0)$ it would be the future and time values would be positive.

This formula can be adjusted as follows:

$$\left|\, t_0 - t \,\right| = \pi\,\sigma^2 \times \left\{ \Phi\left(\frac{p-\mu}{\sigma}\right) + \Phi\left(\frac{\mu}{\sigma}\right) \right\}. \qquad (18)$$

I should like to notice Theory of Relativity reflects the present like a point laying on the time axis. In this point is an observer. On the left it is the past and values are negative, on the right it is the future with positive values. From the point of the geological view it is not important in which of the time interval points the observer is, but the crucial is the absolute value of the geological time.

Very important is to know how large the time error of geological age is. It is possible to use formula (8) for derivation of the time error.

$$-(t + \Delta t) = \pi\,\sigma^2 \times \left\{ \Phi\left(\frac{p_{\max}-\mu}{\sigma}\right) - \Phi\left[\left(\frac{p-\mu}{\sigma}\right) + \frac{\Delta p}{\sigma}\right] \right\}. \qquad (19)$$

Due to formulas (8) and (19) we get the following formula:

$$\left|\Delta t\right| = \pm\,\pi\,\sigma^2 \times \left\{ \Phi\left[\left(\frac{p-\mu}{\sigma}\right) + \frac{\Delta p}{\sigma}\right] - \Phi\left(\frac{p-\mu}{\sigma}\right) \right\}. \qquad (20)$$

If $[(p-\mu)/\sigma] \to 0$ it holds that:

$$\left|\Delta t\right| \to \left|\Delta t_{\max}\right| = \left|\pi \times \sigma^2\right| \;. \qquad (21)$$

In such case it is the maximal error which is observed for $p \to \mu$ and $\Delta p >> \sigma$. If $[(p-\mu)/\sigma] \to 1$ then there holds this condition:

$$\left|\Delta t\right| = \pm\,\pi\,\sigma^2 \times \left\{ \Phi\left[1 + \frac{\Delta p}{\sigma}\right] - \Phi(1) \right\}. \qquad (22)$$

In such case it holds; $|\,\Delta t\,| \to 0$. This case is for $p \to \mu \pm \sigma$, i.e., both $p \to 0$ and $p \to p_{max}$, and together with condition $\Delta p << \sigma$.

4 Derivation of the evolution curve for accelerated sedimentary consolidation

The law of lognormal distribution with positive asymmetry looks like that:

$$f^*(p) = \frac{1}{\sigma\sqrt{2\pi}} \times p^{-1} \times \exp\left\{ -\frac{1}{2} \times \left(\frac{\ln p - \mu}{\sigma}\right)^2 \right\}. \qquad (23)$$

We have to start again from the law describing the velocity of sedimentary consolidation being very similar to the law of lognormal distribution with positive asymmetry.

$$\frac{dp}{dt} = f(p) = \frac{1}{\sigma\sqrt{2\pi}} \times p^{-1} \times \exp\left\{ +\frac{1}{2} \times \left(\frac{\ln p - \mu}{\sigma} \right)^2 \right\}. \qquad (24)$$

As in this case there is not normally distributed p, but ln p; it holds that the characteristic μ is the mean of ln p. We can express the time like that:

$$t = \sigma\sqrt{2\pi} \int_{p_{\max}}^{p} p \times \exp\left\{ -\frac{1}{2} \times \left(\frac{\ln p - \mu}{\sigma} \right)^2 \right\} dp \ldots \text{for } p_{\max} \rangle\, p . \qquad (25)$$

Thanks to substitutions $q = [(\ln p - \mu)/\sigma]$ and $s = q + 2\sigma$ we attain this integral:

$$t = -\pi\sigma^2 \times e^{2(\mu-\sigma^2)} \times \frac{2}{\sqrt{2\pi}} \int_0^{[(\ln p_{\max}-\mu)/\sigma+2\sigma]} \exp\left\{ -\frac{1}{2} s^2 \right\} ds + \\ + \pi\sigma^2 \times e^{2(\mu-\sigma^2)} \times \frac{2}{\sqrt{2\pi}} \int_0^{[(\ln p-\mu)/\sigma+2\sigma]} \exp\left\{ -\frac{1}{2} s^2 \right\} ds . \qquad (26)$$

The integral was separated on two ones. With the help of identical transformation like it was done for normal distribution we have got formula which is depicted like relationship in fig.1.

$$|\, t \,| = \pi\sigma^2 \times e^{2(\mu-\sigma^2)} \times \left\{ \Phi\left(\frac{\ln p_{\max} - \mu}{\sigma} + 2\sigma \right) - \Phi\left(\frac{\ln p - \mu}{\sigma} + 2\sigma \right) \right\}. \qquad (27)$$

Note, please, that concurrently holds:

As $p_{\max} > p$ then $\ln p_{\max} > \ln p$.

The future time is expressed as follows:

$$(t_0 - t) = -\pi\sigma^2 \times e^{2(\mu-\sigma^2)} \times \left\{ 1 + \Phi\left(\frac{\ln p - \mu}{\sigma} + 2\sigma \right) \right\} . \qquad (28)$$

It results from properties of Laplace's Function. It is possible to write:

$\Phi(-q) = -\Phi(q)$, $\Phi(\infty) = +1$ and $\Phi(-\infty) = -1$.

Formula (28) can be adjusted.

$$|\, t_0 - t \,| = \pi\sigma^2 \times e^{2(\mu-\sigma^2)} \times \left\{ 1 + \Phi\left(\frac{\ln p - \mu}{\sigma} + 2\sigma \right) \right\} . \qquad (29)$$

Note, please, yet if p = 0 Laplace's Function is equal to -1 and $|t_0 - t| = 0$. We have also to deduce the formula of the time error.

$$-\left(t+\Delta t\right)=\ \pi\,\sigma^{2}\times e^{2(\mu-\sigma^{2})}\times\left\{\quad\Phi\left(\frac{\ln p_{\max}-\mu}{\sigma}+2\sigma\right)-\Phi\left(\frac{\ln\left(p+\Delta p\right)-\mu}{\sigma}+2\sigma\right)\right\}. \quad (30)$$

Due to formula (30) you will obtain this equation:

$$\left|\Delta t\right|=\ \pm\,\pi\,\sigma^{2}\times e^{2(\mu-\sigma^{2})}\times\left\{\ \Phi\left\{\left(\frac{\ln p-\mu}{\sigma}+2\sigma\right)+\frac{\ln\left(1+\Delta p/p\right)}{\sigma}\right\}-\Phi\left(\frac{\ln p-\mu}{\sigma}+2\sigma\right)\right\}. \quad (31)$$

This formula has again its maximal error which is shifted towards lower values of total porosity on comparison to normal distribution. For both $p \to 0$ and $p \to p_{max}$ and $\Delta p \ll \sigma$ it holds that $|\Delta t| \to 0$.

5 Derivation of the evolution curve for delayed sedimentary consolidation

The law of lognormal distribution has negative asymmetry and looks like this:

$$f^{*}\left(p\right)=\frac{1}{\sigma\sqrt{2\pi}}\times\left(1-p\right)\times\exp\left\{-\frac{1}{2}\times\left(\frac{\ln\left(1-p\right)^{-1}-\mu}{\sigma}\right)^{2}\right\}. \quad (32)$$

The beginning of that is again that differential equation; the law is exponential again and closed to the law of lognormal distribution with negative asymmetry.

$$\frac{dp}{dt}=f\left(p\right)=\frac{1}{\sigma\sqrt{2\pi}}\times\left(1-p\right)\times\exp\left\{+\frac{1}{2}\times\left(\frac{\ln\left(1-p\right)^{-1}-\mu}{\sigma}\right)^{2}\right\}. \quad (33)$$

Here is normally distributed expression $\ln (1-p)^{-1}$. The bygone time has the following shape:

$$t=\sigma\sqrt{2\pi}\int_{p_{\max}}^{p}\left(1-p\right)^{-1}\times\exp\left\{-\frac{1}{2}\times\left(\frac{\ln\left(1-p\right)^{-1}-\mu}{\sigma}\right)^{2}\right\}dp\ \ldots\text{for } p_{\max}\rangle\ p, \quad (34)$$

Due to substitutions like is $m = (1- p)^{-1}$ and $q = [(\ln m – \mu)/\sigma]$ we transform formula looking like formula (35); newly created formula has this form:

$$t=-\,\pi\,\sigma^{2}\times\frac{2}{\sqrt{2\pi}}\int_{0}^{[(\ln(1-p_{\max})^{-1}-\mu)/\sigma]}\exp\left\{-\frac{1}{2}q^{2}\right\}dq+\pi\,\sigma^{2}\times\frac{2}{\sqrt{2\pi}}\int_{0}^{[(\ln(1-p)^{-1}-\mu)/\sigma]}\exp\left\{-\frac{1}{2}q^{2}\right\}dq\,. \quad (35)$$

This integral is separated on two ones and adjusted to reach form of Laplace's Function. It presents the absolute value of the bygone time.

$$|\, t \,| = \pi \sigma^2 \times \left\{ \Phi\left(\frac{\ln\left(1 - p_{\max}\right)^{-1} - \mu}{\sigma} \right) - \Phi\left(\frac{\ln\left(1 - p\right)^{-1} - \mu}{\sigma} \right) \right\}. \qquad (36)$$

Formula (36) is presented in fig.1. For the time remaining when all pore space will have been filled with cement you will get formula as follows; necessary adjustments are the same like it was in the before chapters.

$$|\, t_0 - t \,| = \pi \sigma^2 \times \left\{ \Phi\left(\frac{\ln\left(1 - p\right)^{-1} - \mu}{\sigma} \right) + \Phi\left(\frac{\mu}{\sigma} \right) \right\}. \qquad (37)$$

For derivation of the time error you can use adjusted formula (36).

$$-\left(t + \Delta t\right) = \pi \sigma^2 \times \left\{ \Phi\left(\frac{\ln\left(1 - p_{\max}\right)^{-1} - \mu}{\sigma} \right) - \Phi\left(\frac{\ln\left(1 - p - \Delta p\right)^{-1} - \mu}{\sigma} \right) \right\}. \qquad (38)$$

We receive formula (39) for the time error.

$$|\Delta t| = \pm \pi \sigma^2 \times \left\{ \Phi\left(\left(\frac{\ln\left(1 - p\right)^{-1} - \mu}{\sigma} \right) + \frac{1}{\sigma} \times \ln\left[1 - \left(\frac{p}{1 - p} \right) \times \left(\frac{\Delta p}{p} \right) \right] \right) - \Phi\left(\frac{\ln\left(1 - p\right)^{-1} - \mu}{\sigma} \right) \right\}. \qquad (39)$$

The above formula is characterized with its maximal error when $[(\ln (1-p)^{-1} - \mu)/\sigma] \rightarrow 0$. If $p \rightarrow 0$ or $p \rightarrow p_{max}$ and simultaneously $\Delta p << \sigma$, it holds that $|\, \Delta t \,| \rightarrow 0$.

6 Implement of the beginning moment of sedimentary consolidation and discussion over dimension of time unit

In this chapter there sound two very important questions discussed; where is laid the beginning moment of sedimentary consolidation and what time unit we have to choose.

As for the beginning moment I should like shortly to comment something about sedimentary process. This has two stages; the stage of accumulation and the stage of consolidation.

Accumulation of sediments is characterized by condition that quantity of water prevails over quantity of sedimentary material. The stage starts from almost clean water and finishes by creation of unconsolidated quick sediments like mud and sand are. From view of total porosity it is interval $0.5 < p < 1$, where the beginning moment of accumulation is very close to clean water, i.e. $p \rightarrow 1$, however, it is not directly $p = 1$.

Consolidation of sediments begins at the moment when unconsolidated sediment is transforming in the rock. This means; quantity of water is lower than quantity of sedimentary material, i.e., $0 < p < 0.5$. The beginning of consolidation is for $p = 0.5$, when a liquid quick sediment is hardening in the solid dead one and this is rock yet. This stage is influenced by many various factors which positively and negatively act on consolidation.

Selection that $p_{max}= 0.5$ presents a difference in comparison to the time determined by method of radioactive decay of elements. This mentioned time goes ahead the stage of consolidation and its beginning is rather closer to $p_{max}\rightarrow 1$. Therefore it is very probable that new method of determination of rock age for sediments will yield data of younger age.

And now, I should like to remember that among rocks there exist also such rocks having $p > 0.5$ what presents their grains are separated. They are the flowing quick sands when the single grains are separated one another by thin water film. The film serves like a lubricant. These beds are under high pressure and after opening of such bed the sands very rapidly move themselves. Therefore they are very dangerous, too.

The next question is selection of time unit. The enumerated values of t_0 when $p = 0$ are following: $t_0^{(a)} = 0.0150925$, $t_0^{(n)} = 0.0333366$ and $t_0^{(d)} = 0.0605317$. It is after tab.2. Index (a) presents accelerated evolution, index (n) normal evolution and index (d) delayed evolution. All three values are expressed in the same time unit which is very low. Such unit can be only one milliard of year, other lower the time unit it cannot be. These are not tens, hundreds or thousands of years. It tends me to that the proved inverse relation by formulas (9) and (13) being between total porosity and the bygone time. In the contrary to very low values of porosity, $0 \leq p \leq 1$, and values of Laplace's Function, $-1 \leq \Phi(q) \leq +1$, there are very high values of the bygone time. The time unit can be only one million or one milliard of years. The million like a time unit you can exclude, because is very small, if you control numeric values. But, the numeric values of the bygone time are well acceptable for the remaining time unit, one milliard. We can transform them from milliards on millions of years.

If we multiply this unit by 10^3 we get the age of rocks in millions of years. That is why you can write; $t_0^{(a)} = 0.0150925\times10^9$ yrs = 15.0925×10^6 yrs, $t_0^{(n)} = 0.0333366\times10^9$ yrs = 33.3366×10^6 yrs and $t_0^{(d)} = 0.0605317\times10^9$ yrs = 60.5317×10^6 yrs. It makes us adjust the before derived formulas into form like follows.

For accelerated evolution it holds:

$$|\,t\,| = \pi\sigma^2\times e^{2(\mu-\sigma^2)}\times\left\{\Phi\left(\frac{\ln p_{\max}-\mu}{\sigma}+2\sigma\right)-\Phi\left(\frac{\ln p-\mu}{\sigma}+2\sigma\right)\right\}\times10^3, \qquad (40)$$

$$|\,t_0-t\,| = \pi\sigma^2\times e^{2(\mu-\sigma^2)}\times\left\{1+\Phi\left(\frac{\ln p-\mu}{\sigma}+2\sigma\right)\right\}\times10^3, \text{ and} \qquad (41)$$

$$|\Delta t| = \pm\pi\sigma^2\times e^{2(\mu-\sigma^2)}\times\left\{\Phi\left\{\left(\frac{\ln p-\mu}{\sigma}+2\sigma\right)+\frac{\ln(1+\Delta p/p)}{\sigma}\right\}-\Phi\left(\frac{\ln p-\mu}{\sigma}+2\sigma\right)\right\}\times10^3. \qquad (42)$$

For normal evolution this is valid:

$$|\,t\,| = \pi\,\sigma^2 \times \left\{ \Phi\left(\frac{p_{\max} - \mu}{\sigma}\right) - \Phi\left(\frac{p-\mu}{\sigma}\right) \right\} \times 10^3 , \quad (43)$$

$$|\,t_0 - t\,| = \pi\,\sigma^2 \times \left\{ \Phi\left(\frac{p-\mu}{\sigma}\right) + \Phi\left(\frac{\mu}{\sigma}\right) \right\} \times 10^3 , \text{ and} \quad (44)$$

$$|\Delta t\,| = \pm\,\pi\,\sigma^2 \times \left\{ \Phi\left[\left(\frac{p-\mu}{\sigma}\right) + \left(\frac{\Delta p}{p}\right) \times \frac{p}{\sigma} \right] - \Phi\left(\frac{p-\mu}{\sigma}\right) \right\} \times 10^3 . \quad (45)$$

For delayed evolution it holds:

$$|\,t\,| = \pi\,\sigma^2 \times \left\{ \Phi\left(\frac{\ln(1 - p_{\max})^{-1} - \mu}{\sigma}\right) - \Phi\left(\frac{\ln(1-p)^{-1} - \mu}{\sigma}\right) \right\} \times 10^3 , \quad (46)$$

$$|\,t_0 - t\,| = \pi\,\sigma^2 \times \left\{ \Phi\left(\frac{\ln(1-p)^{-1} - \mu}{\sigma}\right) + \Phi\left(\frac{\mu}{\sigma}\right) \right\} \times 10^3 , \text{ and} \quad (47)$$

$$|\Delta t\,| = \pm\,\pi\,\sigma^2 \times \left\{ \Phi\left(\left(\frac{\ln(1-p)^{-1} - \mu}{\sigma}\right) + \frac{1}{\sigma} \times \ln\left[1 - \left(\frac{p}{1-p}\right) \times \left(\frac{\Delta p}{p}\right)\right]\right) - \Phi\left(\frac{\ln(1-p)^{-1} - \mu}{\sigma}\right) \right\} \times 10^3 . \quad (48)$$

These all formulas had been used for enumeration of tab.1.

7 Discussion over tables and depicted plots

Tab.1 presents results of enumeration after formulas for | t | and | Δt | together. This table premises that Δp = ± 0.01. The course of both variables is very interesting; both predict accuracy of determination of the rock age.

The variable remarked as | Δt | has for total porosity its maximal value presenting the maximal time error. Towards both borders of the porosity interval the variable tends to zero. The variable remarked as | t | has its highest value for p = 0, whereas, for p = 0.5 its value is zero.

As for ancient rocks having very low porosity there is the bygone time high, but, the time error tends to zero; the relative error (Δt / t) is declining to zero. The recent rocks having high porosity have the time error also tending to zero, however, the bygone time is tending to zero, too. And just this makes that (Δt / t) is growing up. A certain exception can be only the accelerated evolution. Conclusion of that is the ancient rocks are determined with higher accuracy than the recent rocks.

After tab.2 there are depicted three evolution curves like function p = f (t). All depicted curves are very similar to the curve of declivity. They are depicted in fig.1. We can distinguish three sections of curve.

Tab. 1 The data of the bygone time and the time errors for accelerated, normal and delayed evolutions

p	Δp/p	|t(a)| ± Δt(a)	Δt(a)/t(a)	|t(n)| ± Δt(n)	Δt(n)/t(n)	|t(d)| ± Δt(d)	Δt(d)/t(d)
		[10^6 yrs]		[10^6 yrs]		[10^6 yrs]	
0.00	∞	15.0925±0.0000	0.0000	33.3366±0.3699	0.01110	60.5317±0.5864	0.0097
0.01	1.0000	15.0788±0.4008	0.0266	32.9672±0.4662	0.01410	59.7966±0.7078	0.0118
0.02	0.5000	14.6781±1.2694	0.0865	32.5005±0.5780	0.01780	58.9089±0.8468	0.0144
0.03	0.3333	13.4087±1.8710	0.1395	31.9219±0.7053	0.02210	57.8486±0.9985	0.0173
0.04	0.2500	11.5373±2.0136	0.1745	31.2173±0.8430	0.02700	56.5964±1.1678	0.0206
0.05	0.2000	9.5245±1.8612	0.1954	30.3748±0.9875	0.03250	55.1348±1.3448	0.0244
0.10	0.1000	2.9234±0.6340	0.2169	23.9541±1.6773	0.07000	44.3532±2.2573	0.0509
0.15	0.0667	0.8854±0.1804	0.2037	14.9878±1.8046	0.12040	28.8352±2.6162	0.0907
0.20	0.0500	0.2934±0.0551	0.1878	6.9178±1.2300	0.17780	13.6324±1.9565	0.1435
0.25	0.0400	0.1062±0.0181	0.1704	2.239±0.5333	0.23820	4.1664±0.8620	0.2069
0.30	0.0333	0.0411±0.0068	0.1655	0.4917±0.1462	0.29730	0.7213±0.2022	0.2803
0.35	0.0286	0.0164±0.0023	0.1402	0.0718±0.0258	0.35930	0.0606±0.0202	0.3333
0.40	0.0250	0.0064±0.0008	0.1250	0.0069±0.0034	0.49280	0.0021±0.0025	1.1905
0.45	0.0222	0.0020±0.0000	0.0000	0.0004±0.0000	0.00000	0±0.0000	
0.50	0.0200	0±0.0000		0±0.0000		0±0.0000	
		μ = -2.1335		μ = 0.1380		μ = 0.1524	
		σ = 0.5782		σ = 0.0740		σ = 0.0897	
		Δp = ± 0.01		Δp = ± 0.01		Δp = ± 0.01	

- The stage of compaction: the relatively short and fast, almost precipitous, the compaction being characterized also by selection of the structure grain system: cubic, tetragonal and hexagonal. Section of curve of this stage varies between an exponential and logarithmic.
- The stage of compression: the long-acting and slow compression of the selected structure system; it has linear section and the condition that p = 0 holds for finite time. In this stage the rock slowly reduces its volume of pores; the grains are mutually shifting yet. Compaction

Tab. 2 Data of relations t = f (p) for accelerated, normal and delayed evolution

p	|t (a)|	|t (n)|	|t (d)|
	[10^6 yrs]	[10^6 yrs]	[10^6 yrs]
0.00	15.0925	33.3366	60.5317
0.01	15.0788	32.9672	59.7966
0.02	14.6781	32.5005	58.9089
0.03	13.4087	31.9219	57.8486
0.04	11.5373	31.2173	56.5964
0.05	9.5245	30.3748	55.1348
0.10	2.9234	23.9541	44.3532
0.15	0.8854	14.9878	28.8352
0.20	0.2934	6.9178	13.6324
0.25	0.1062	2.239	4.1664
0.30	0.0411	0.4917	0.7213
0.35	0.0164	0.0718	0.0606
0.40	0.0064	0.0069	0.0021
0.45	0.002	0.0004	0
0.50	0	0	0

and compressions are most likely two diverse phases of the only pressure process. The changes of porosity are completed by starting cementation activity which, with time, gains bigger significance and all process shortens.

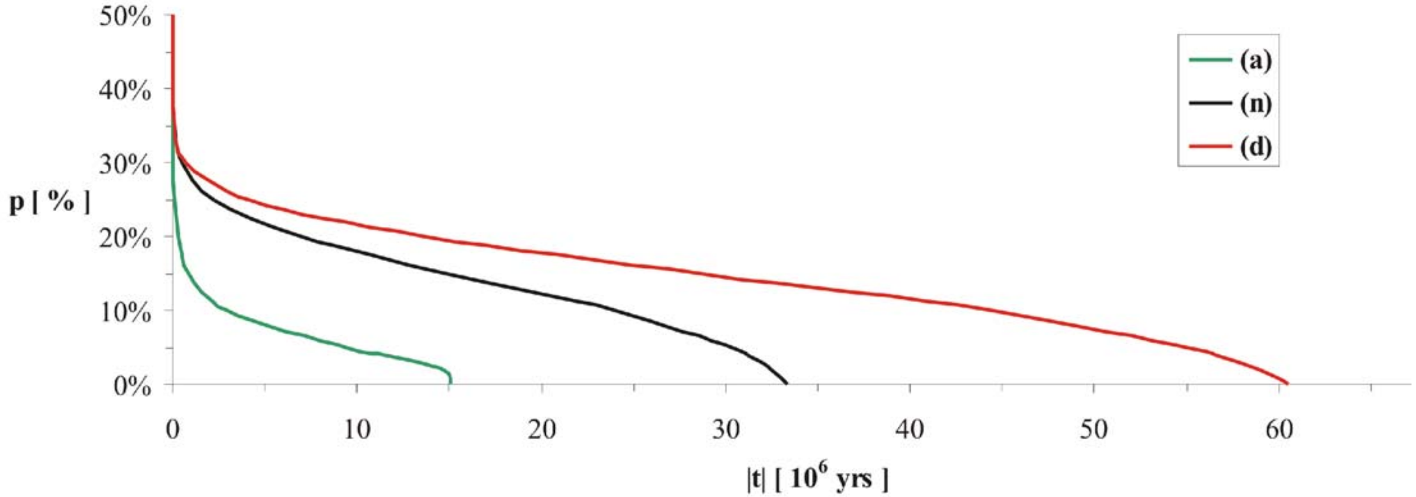

Fig. 1 Relations of type p = f (t) for accelerated, normal and delayed evolution

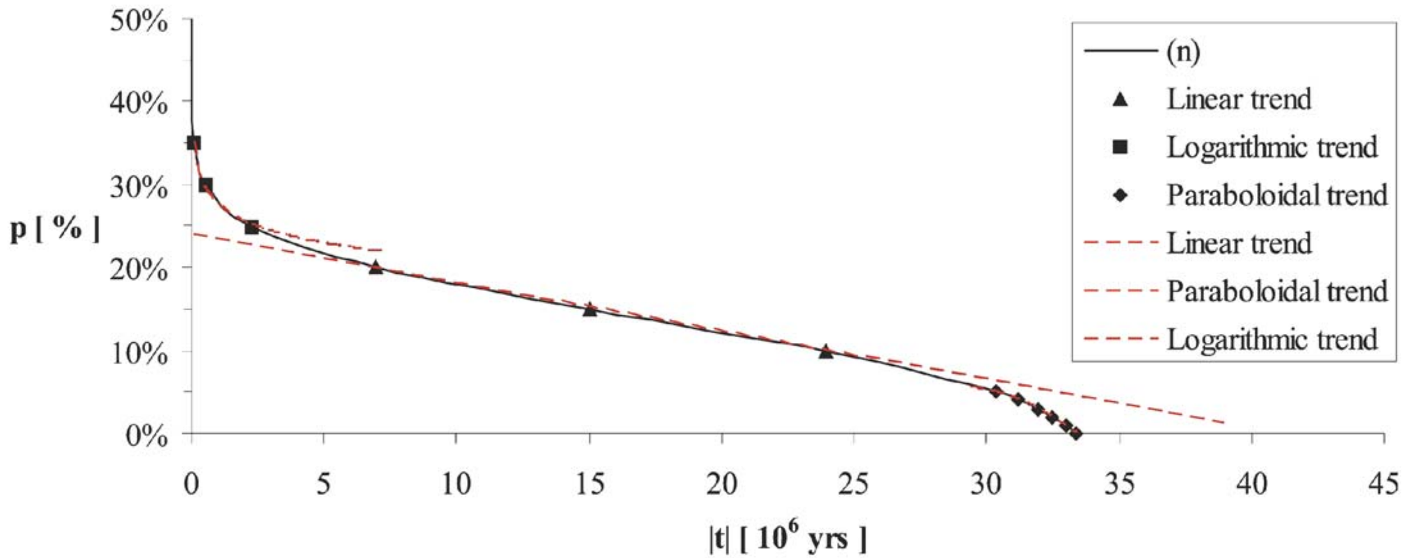

Fig. 2 Basic three stages of the consolidation process for normal evolution

- The stage of cementation: final stage when the structure of grains has almost made yet and all changes are made by the rock cementation only. This section of curve is likely parabolic. The mentioned stage is well visible for lower porosity, lower than p = 0.15. It is because; the streaming water is not, thanks to big capillary forces, such strong to be able to strip and to dissolve the cement layers creating among grains. But if sand/sandstone has higher porosity, higher than p = 0.15, the streaming water such force has and is able every cement layer to destroy. And that is why we do not observe a parabolic curvature there. Strictly speaking, cementation under higher porosity has no chance to assert itself.

All three stages have its only final value of time for p = 0. It is depicted in fig.2. Thanks to relation $|t_0 - t|$ we are able to determine not only the bygone time, but also the remaining time towards future. This relation is the next of the time characteristics.

The used characteristics μ and σ for tab.1 and tab.2 are from the paper RYŠAVÝ (2004) and they are following: $\mu^{(a)} = -2.1335$, $\sigma^{(a)} = 0.5782$, $\mu^{(n)} = 0.1380$, $\sigma^{(n)} = 0.0740$, $\mu^{(d)} = 0.1524$ and $\sigma^{(d)} = 0.0897$.

8 Dynamics of process

Even if it seems the process of compaction to be static, it is not true. It is infinitely slow dynamic process going on towards future. We have to take depiction of the time curve like a snap in this time. Such snap says what evolution has been from the past up to recent time. Thanks to dynamics of the process the new snap made some millions later can be completely other; normal evolution can be replaced by lognormal one, the shape of the time curve can be too different. This is important to keep in mind.

The diagenesis of sediments it is all set of processes acting in sediments. The first it is mechanic consolidation of material and the following compression with weight of sediments later deposited. Here we can expect those differences being between normal and lognormal distribution. In the areas of anomaly movements of blocks, namely the vertical ones, there act forces tending either upwards against the weight of upper sediments (the delayed evolution) or in downwards as the mentioned weight (the accelerated evolution). However, this is not the only factor influencing the type of evolution.

It is, too, the cementation process, the process of recrystalisation, when aragonite is changed in calcite, further, dolomitisation of limestone being bound with the porosity changes, creation of crystals in fissures and folding planes and, of course, decalcification of cement when aragonite/calcite cement is dissolved with acid water. The next diagenesis processes we can anticipate only and some of them we do not know up to now. However, all the processes act in common and their result determines the type of evolution, though for longnormal distribution some of them have higher influence than the remaining.

9 Classification of geological periods after the rock age

Due to derived formulas of the bygone time we are able to make the curve of the apparent time with the borehole depth. If you admit that the registered signal has global and local components, you will have also to specify the domain of their activities. The global one is dependent on the depth, it is a function of the part of signal with depth, and this component is determined with all minimal values of the registered signal. The trend curve of all these data presents the global component and it is the curve of the true time.

The local component is not dependent on the borehole depth. This component is influenced by local factors acting on the registered signal. In practice it is the difference between the common signal formed with both components and the global one.

Tab. 3 Data of the apparent time for fig. 3. The bold data are for the curve of trend

H	p	|t|	H	p	|t|
[m]		[10^6 yrs]	[m]		[10^6 yrs]
101	0.28	0.9462	126	0.12	20.5081
102	0.26	1.7066	127	0.11	22.2766
103	**0.35**	**0.0723**	128	0.13	18.6846
104	0.27	1.2817	129	0.09	25.5195
105	**0.31**	**0.3458**	130	0.08	26.9542
106	0.2	6.9175	131	0.04	31.2172
107	0.22	4.6071	132	0.07	28.2462
108	**0.23**	**3.6781**	133	0.17	11.4488
109	0.16	13.1812	134	0.16	13.1812
110	0.15	14.9858	135	0.11	22.2766
111	0.17	11.4488	**136**	**0.21**	**5.6874**
112	0.21	5.6874	137	0.19	8.2955
113	**0.22**	**4.6071**	138	0.14	16.8335
114	0.18	9.8111	139	0.13	18.6846
115	0.17	11.4488	140	0.07	28.2462
116	0.11	22.2766	141	0.08	26.9542
117	0.12	20.5081	142	0.03	31.9226
118	0.1	23.954	143	0.04	31.2172
119	0.05	30.3743	144	0.06	29.3868
120	0.11	22.2766	145	0.12	20.5081
121	0.06	29.3868	146	0.11	22.2766
122	0.07	28.2462	147	0.1	23.954
123	0.03	31.9226	148	0.11	22.2766
124	0.09	25.5195	149	0.13	18.6846
125	0.08	26.9542	150	0.1	23.954

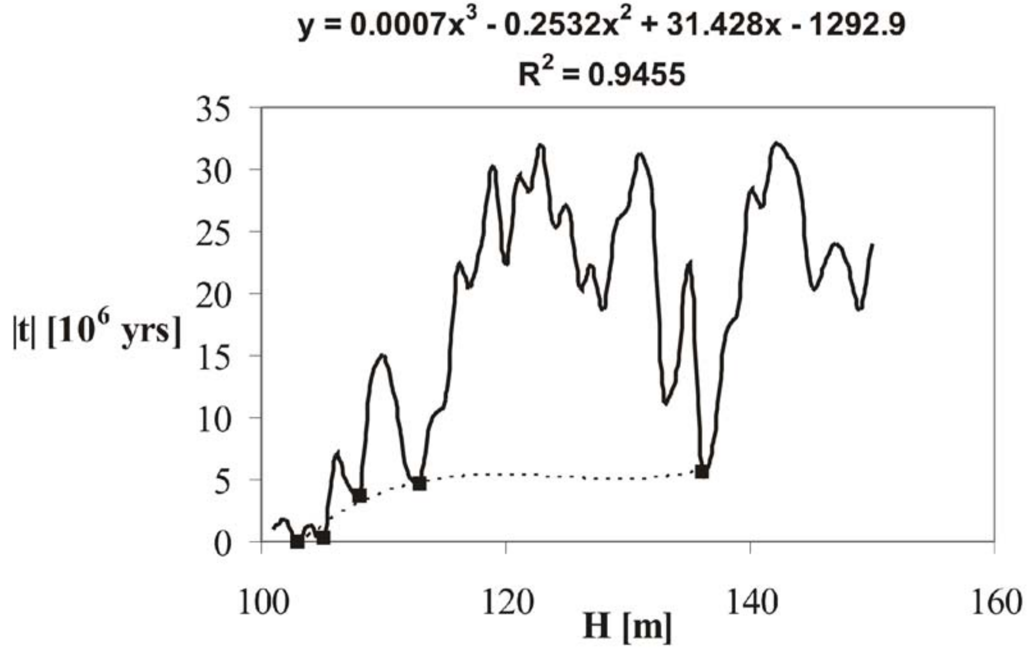

Fig. 3 The curve of the apparent time after tab.3 with the global component

The global component we identify with shaly fraction, whereas, the local one is subjoined to sandy/calcareous fraction. The shaly fraction is plastic and the sedimentary consolidation in plastic rocks proceeds in other way than it is in hard rocks being sandy or calcareous. Here is only stage of compression; stage of cementation does not assert oneself.

For local component it is characteristic that the pores of sandy fraction because they are middle and big, are easy filled with cement and that is why that total porosity of sands and sandstones is often so low. This component increases the apparent time. It is false information. The stage of cementation has decisive influence for this component.

The global component being formed with plastic shales has its pores tiny, but, resisting to cementation. The shales remain plastic practically for all period of consolidation until they will be changed into slates. It is the last phase of their consolidation.

Insertion of the trend curve of the global component is easier there where the intervals of shales are well-visible. Fast changing of shales and sands/carbonates make that more difficult, but it is possible too. This all is only about construction of the shale-line in the time domain; you know it very well for the SP- method, for example.

The curve of the local component is not important for interpretation of the rock age. It seems it is an effect of the hard non-plastic ingredient of rocks being transformed into the time domain. It carries also information; maybe, it can be about degree of cementation of the pore space.

Tab.3 carries data of a fictive borehole. Data of porosity are random selected by the random number generator. The depth is fictive too. Fig.3 presents the curve of the apparent time made thanks to tab.3. There was used transformation defined like | t | = f (p) for the normal sedimentary consolidation. Note, please, there are data in bold letters. They are those having been used for construction of the trend curve presenting the shaly global component. This is depicted like dashed line and it is the polynomial relation of the third degree. It presents the curve of the true time as a mark of only compaction and compression.

The curve of the global component is convenient for classification of the geological periods and for construction of the borehole section for the above periods after well-logging data only. However, you need for that the geological table of age of those periods. The rough distribution of periods is in tab.4 there. It is after SHERIFF (1973). Of course, the more detailed distribution of table is not excluded.

Tab. 4 The absolute age of the geological periods

Period	**Age [10^6 yrs]**	**Period**	**Age [10^6 yrs]**
Archeozoic	2000-4600	Jurassic	136-189
Proterozoic	570-1999	Cretaceous	65-135
Cambrian	500-569	Paleocene	54-64
Ordovician	430-499	Eocene	38-53
Silurian	395-429	Oligocene	26-37
Devonian	345-394	Miocene	7-25
Carboniferous	280-344	Pliocene	2.5-6
Permian	225-279	Pleistocene	0.005-2.49
Triassic	190-224	Recent	0-0.0049

10 Conclusions

On the base of analysis having been made here are these conclusions:

- With the help of the evolution curves written down in the form of differential equation and describing relationship between total porosity and geological time it is possible exactly to express the age of sedimentary rocks;
- Function f (p) has exponential progression; the function is closed to functions of statistic distribution f*(p) and uses statistic characteristics μ and σ;
- This age is calculated from the beginning of sedimentary consolidation of rocks. Therefore the above age must be younger than the age calculated after the method of decay of radioactive elements;
- We distinguish three different types of sedimentary consolidation: the accelerated evolution, the normal evolution and, finally, the delayed evolution. Each of them has its own characteristic distribution;
- The rocks having identical porosity have three potential geological ages; on the other side, the rocks having identical age can have three different values of total porosity yet thanks to three different evolutions;
- The evolution curves of all three evolutions are very similar to. They have its own final time for p = 0. Thanks to them it is possible to define not only the bygone time, but also, the remaining time up to that moment for p = 0;
- Process of consolidation depicted like curve p = f (t) has three stages: compaction, compression and cementation;

- For each of evolutions we can determine the absolute time error. Due to the relative error it is possible to estimate accuracy of time determination. There are more exactly determined the ancient rocks than the recent ones;
- It is possible to make the borehole section of the bygone geological periods with the help of tab.4 and the curve of the true time with the borehole depth.

References

KETTNER R. *Všeobecná geologie, 2*, ČSAV, Praha, 1957, p. 270

MIAL and READING,H.G. (ed.) *Sedimentary Environments and Facies,* Blackwell Scientific Publications, Oxford, 1978, p. 557

REINECK,H. E. and SINGH, I. B. *Depositional Sedimentary Environments*, Springer-Verlag, Berlín, 1975, p. 439

REKTORYS et al. *Přehled užité matematiky*, 2. opravené vydání, SNTL, Praha, 1968, p.171−174

RYŠAVÝ F. Analysis of the Diagenesis Development of the Sedimentary Basin through Total Porosity of Sediments on Basis of Well-Logging Data*, Acta Universitatis Carolinae – Geologica 2004, 1 – 4*, 2004, p. 1 – 8.

SERRA,O. *Fundamentals of Well-log data Interpretation. 2. The interpretation of logging data,* Elsevier, Amsterdam, 1986, p. 684

SERRA,O. and SULPICE,L. Sedimentological analysis of shale-sand series from well logs. – SPWLA. *16th Ann. Log.Symp.Trans., paper W*, 1975

SERRA,O.and SERRA,L. *Well Logging and Geology.* – Serralog, Calvados, France, 2003, p. 436

SHERIFF R.E. *Encyclopedic Dictionary of Exploration Geophysics*, Society of Exploration Geophysics, Tulsa, Oklahoma, 1973

ŠKRÁŠEK J., TICHÝ Z. *Základy aplikované matematiky* 2, SNTL, Praha, 1986, p. 446−449

ŠKRÁŠEK J., TICHÝ Z. *Základy aplikované matematiky* 3, SNTL, Praha, 1990, p. 111−112

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