METRIC SPACES AND QUALITY CONTROL OF ROTARY DRILLING PROCESS

METRICKÉ PRIESTORY A RIADENIE KVALITY PROCESU ROTAČNÉHO VŔTANIA

Igor Leššo [1], Patrik Flegner [2], Katarína Feriančiková [3]

Abstract

The paper is oriented toward research and development of the possibilities of using the theory of abstract metric spaces in the solution of selected problems in geotechnology. The notion of „metric space“is introduced in functional analysis as a special chapter of so-called modern mathematics. It is a mathematical abstraction of the classical 3-dimentional Euclid space. The article investigates the possibilities of constructing such abstract spaces, where the elements (points) are geophysical signals or multidimensional data connected with the solution of specific problem in the area of geotechnology. This approach enables to solve many problems as problems of classification of objects or states of processes as elements of the space represented by its specific signals or data. Application of metric spaces subsequently opens up the possibility of application of one of the methods of artificial intelligence.

Abstrakt

Článok je zameraný na výskum možností využitia teórie abstraktných metrických priestorov pri riešení vybraných problémov v geotechnike. Pojem „metrický priestor“ zavádza funkcionálna analýza ako špeciálna kapitola tzv. modernej matematiky. Ide o matematickú abstrakciu klasického trojrozmerného Euklidovho priestoru. Príspevok skúma možnosti konštrukcie takýchto abstraktných priestorov, kde by jeho prvkami (bodmi) boli geofyzikálne signály, resp. viacrozmerné dáta spojené s riešením konkrétneho problému z oblasti geotechnológií. Tento prístup umožní riešiť mnohé problémy ako problémy klasifikácie objektov alebo stavov procesu ako prvkov priestoru, reprezentovaných svojimi špecifickými signálmi alebo dátami. Aplikácia metrických priestorov otvára následne možnosť nasadenia niektorej z metód umelej inteligencie.

Keywords
metric space, Hilbert space, geophysical signal, classification, vector quantization of space

Kľúčové slová
metrický priestor, Hilbertov priestor, geofyzikálny signál, klasifikácia, vektor kvantifikácie priestoru

1 Introduction

Many problems in the area of Geoscience, but also problems in engineering geology, geophysics, seismology, etc., are solved with methods based on measurement, processing and evaluation of various geophysical signals (Krepelka, 2008), (Pandula, 2010). Such geophysical signals could be for example seismic recordings, responses of structures to artificial or natural excitation, seismological recordings of earthquakes, signals of mechanical oscillation in a rock massif in seismic tomography, etc.

These methods of evaluation of signals are frequently based on their mutual comparison or on the comparison of measured signals with certain template signals (norm). In some cases the dynamics of changes of certain specific signal is monitored in repeated measurements over a period of time. The efficiency of these methods can be increased if we can find an exact way of mutual comparison of signals. Very interesting and often highly efficient possibilities are offered by functional analysis (Taylor, 1973), namely the theory of abstract metric spaces. Functional analysis identifies a function, satisfying certain basic properties, with a point (vector) in space and analogously to the classical Euclidean 3-dimensional space it defines different properties between points of the space from the viewpoint of topology and geometry.

For example, it is possible to define, in a topological and geometrical sense, the distance between two functions in a space which expresses their distinction, similarly the angle subtended by these functions as vectors in the space. It is a property that only identical functions can have zero distance, zero angles can be subtended only by functions that differ only by being a constant multiple of each other (collinear vectors). A special meaning in these abstract vectors has the algebraic operation of scalar product of two vectors in space which takes on the value 0 for two mutually perpendicular vectors. In the case of vectors as functions, this scalar product is zero for so-called orthogonal (mutually perpendicular) or orthonormal functions.

The utilization of abstract spaces for the solution of problems in which measured signals are used can be divided into two levels: 1. utilization of real finite-dimensional Euclidean space, 2. utilization of complex infinite-dimensional Hilbert space (Leššo, VEGA 2009).

2 Utilization of abstract Euclidian-type space

The first possibility is frequently used in modern methods of technical diagnostic, but also in methods of artificial intelligence and is based on utilization of finite-dimensional abstract space of Euclidean type $E_N \equiv R^N$. In this case, from each realization of the compared signals a group N of symptoms is computed (extracted), each of which mostly takes on a real value. Then each of the compared signals is represented by a symptom vector $\mathrm{p} = (p_1, p_2, ..., p_N) \in E_N$. Mutual comparison of signals is then performed based on their mutual distance $\rho_{\mathrm{e}}(\mathrm{p}_i, \mathrm{p}_j) \in R^+$ and based on the size of the subtended angle $\varphi(\mathrm{p}_i, \mathrm{p}_j)$:

$$\rho_{\mathrm{e}}(\mathrm{p}_i, \mathrm{p}_j) = \sqrt{\sum_{k=1}^{N} (p_{ik} - p_{jk})^2}, \qquad (1)$$

$$\varphi(\mathrm{p}_i, \mathrm{p}_j) = \arccos \frac{\langle \mathrm{p}_i, \mathrm{p}_j \rangle}{\|\mathrm{p}_i\| \|\mathrm{p}_j\|}, \; 0 \le \varphi \le \frac{\pi}{2}. \qquad (2)$$

For the norms of the symptom vectors we have:

$$\|\mathrm{p}\| = \sqrt{\sum_{k=1}^{N} p_k^2}, \qquad (3)$$

for the scalar product of a pair of symptom vectors the following holds:

$$\langle \mathrm{p}_i , \mathrm{p}_j \rangle = \sum_{k=1}^{N} p_{ik} p_{jk} . \qquad (4)$$

So an interpretation of the signal in such a Euclidean N – dimensional linear algebraic space requires suitable choice of algorithms for the extraction of individual symptoms. The choice of these algorithms determines the measure of mutual differentiability of signals as points in the abstract symptom space $E_N \equiv R^N$.

3 Utilization of abstract Hilbert-type space

A second possibility is less utilized and belongs among higher methods of signal processing. It is based on the utilization of infinite-dimensional abstract Hilbert space H (Naylor, 1981). The fundamental property of the Hilbert space is its set-point structure which makes it possible for a point in space to represent an entire realization of the signal in the form of a continuous bounded function of time or some other independent variable. Moreover, it can be a complex function or complex signal since the coordinates in Hilbert space are complex numbers. Therefore it is the space $H \equiv C^\infty$. For an implementation of the signal as a point in Hilbert space we can use two types of Hilbert space that differ in their set-point structure.

A space of the type $L_p \langle 0, T \rangle$ is a set of all possible complex or real functions continuous and bounded in the interval $\langle 0, T \rangle$. Then the signal $x(t)$ considered as a point in space can be written as follows:

$$x = \left(x(t), t : 0 \to T \right) \in L_p \langle 0, T \rangle . \qquad (5)$$

A space of the type l_p is a set of all possible infinite sequences of complex or real numbers and the signal $x(t)$ considered as its element in the time interval $\langle 0, T \rangle$ can be written as follows:

$$x = \left(x_1 , x_2 , ... \right) \in l_p . \qquad (6)$$

Relation (6) corresponds more to the sampled signal in digital processing, while relation (5) corresponds to an analogue signal before its digitization. In the following text we use notation (5).

Similar to the case of finite-dimensional Euclidean space $E_N \equiv R^N$, also in the case of Hilbert space topological and geometrical relations hold between signals as points in space. For the distance of two signals $x_i(t), x_j(t) \in L_p\langle 0,T\rangle$ we have:

$$\rho\left(x_i, x_j\right) = \left(\int_0^T \left|x_i(t) - x_j(t)\right|^p \mathrm{d}t\right)^{1/p}, 1 \le p < \infty, \tag{7}$$

for the angle subtended by two signals as vectors the following holds:

$$\varphi = \arccos \frac{\left\langle x_i, x_j \right\rangle}{\left\|x_i\right\|_2 \left\|x_j\right\|_2} . \tag{8}$$

For the L_2 norms of signals as vectors we have:

$$\left\|x\right\|_2 = \left(\int_0^T \left|x(t)\right|^2 \mathrm{d}t\right)^{1/2}, \tag{9}$$

for the scalar product we have:

$$\left\langle x_i, x_j \right\rangle = \int_0^T x_i(t) x_j^*(t) \mathrm{d}t, \tag{10}$$

where $\overset{*}{x}_i(t)$ is the complex-conjugate number to the value of the function $x_i(t)$.

The algebraic structure of Hilbert space makes it possible to express (expand) the same vector as a point of the space relative to different orthogonal or orthonormal bases. Specifically it is the general form of so-called Fourier series:

$$x = \sum_{k=0}^{\infty} x_k b_k \,, \qquad (11)$$

where x is the vector expressed in the original basis, coefficients x_k, *and* k=0, 1... are the so-called Fourier coefficients of the expansion (spectrum) and they are the coordinates of the vector x relative to the basis $b = (b_1, b_2, ..., b_k, ...)$. For the coefficients of the expansion we have:

$$x_k = \langle x, b_k \rangle = \int_0^T x(t) \overset{*}{b}_k(t) \mathrm{d}t \,. \qquad (12)$$

The original basis of the read-in signal is the basis of an infinite sequence of unit functions, analogously to the orthonormal basis of the Euclidean space.

In applications, the expansion of the vector $x \in H$ into a Fourier series (11) is mostly done relative to the orthogonal harmonic functions in the exponential form $\left\{ \mathrm{e}^{\mathbf{i}\omega_0 t}, \mathrm{e}^{\mathbf{i}\omega_1 t}, ..., \mathrm{e}^{\mathbf{i}\omega_k t}, ... \right\}$. The coefficients x_k, k=0, 1, 2, of this expansion are given, on the basis of (12), by the relation $x_k = \left\langle x, \mathrm{e}^{\mathbf{i}\omega_k t} \right\rangle = \int_0^T x(t) \mathrm{e}^{-\mathbf{i}\omega_k t} \mathrm{d}t \equiv \hat{F}_k = \left|\hat{F}_k\right| \mathrm{e}^{\mathbf{i}\varphi}$, where the symbolic notation of these coefficients $\hat{F}_k$ says that they are complex numbers (complex amplitudes of harmonic components of the signal). In practical solutions these coefficients of the signal spectrum are calculated for finite frequency resolution with the well-known fast Fourier transform FFT or DFT.

Expression of the signal as a continuous bounded function of time in the interval $\langle 0,T \rangle$ can be done in two equivalent ways:

- similar to (5) as a continuous function of time $x = \left(x(t), t : 0 \to T \right) \in L_p \langle 0,T \rangle$,
- in a modified version of (5) by using its spectrum, in our case the frequency spectrum as
- function $x = \left(x(t) = \frac{1}{2\pi} \int_0^{\infty} \hat{F}(\mathbf{i}\omega) \mathrm{e}^{\mathbf{i}\omega t} \mathrm{d}\omega,\ t : 0 \mapsto T \right) \in L_p \langle 0,T \rangle$, where the function values are calculated from the spectrum of the signal by using the functional for the inverse Fourier transform. In this case the expression $\hat{F}(\mathbf{i}\omega)$ represents continuous complex frequency spectrum of the signal as a continuous analogue of the above-mentioned discrete spectrum $\hat{F}_k(\mathbf{i}\omega),\ k = 0,1,2,...$

The above equivalence of the expressions of the signal in time and frequency domains means, from the viewpoint of Hilbert spaces, that the geophysical signal *x* can be defined from the viewpoint of functional analysis as a point (vector) of Hilbert space of type L_p, while its coordinates can be the values of the signal for $t : 0 \mapsto T$, or complex values of its continuous frequency spectrum $\hat{F}(\mathbf{i}\omega)$ for $\omega : 0 \mapsto \infty$. Thus in both cases it is the same vector of the space *H*, the difference being in the orthogonal basis used. For these two representations of the same signal to be typographically distinguished, it is possible to use different size of letters, so that we have: $x = \left(x(t), t : 0 \mapsto T \right) \in L_p \langle 0,T \rangle$ and $X = \left(\hat{F}(\mathbf{i}\omega), \omega : 0 \mapsto \infty \right) \in L_p \langle 0,\infty)$.

The problem of the interval $\langle 0,\infty)$ not being closed from the right is not treated here since in digital processing of signals this mathematical flaw disappears.

4 Comparison of geophysical signals in symptom space of type E_N

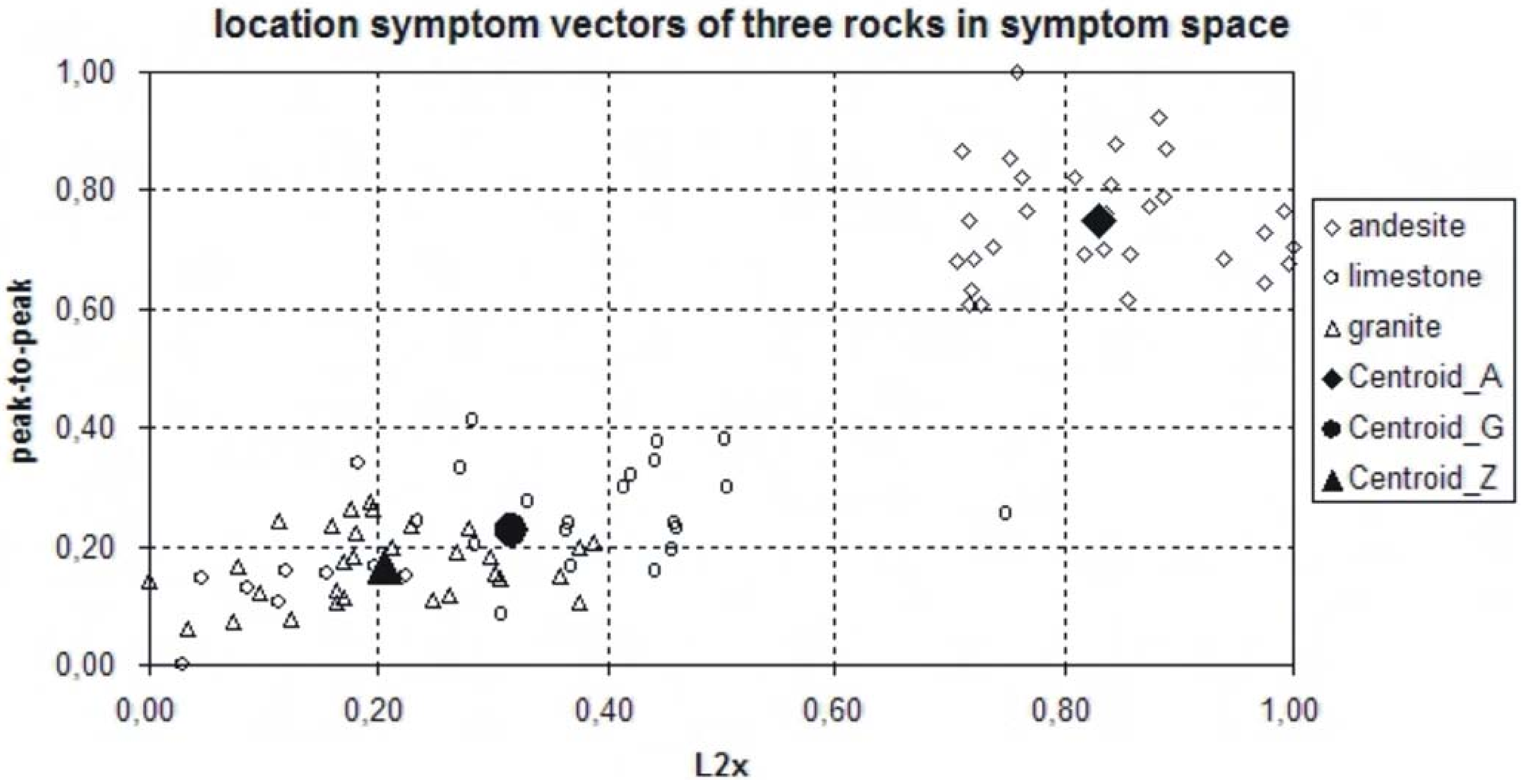

Fig.1 Symptom vectors extracted from realizations of the vibration signal from separation of three rocks in symptom space E_2

One of the areas of geotechnology where abstract mathematical spaces can be utilized is efficient control of the process of rotary drilling of rock massif, where the signal of concurrent vibro-acoustic emissions can be used as an integrating source of information about the current state of the drilling process. The scientific research in this area is oriented toward classification of rock separation and their sorting into common classes. Sorting of the rock currently being separated into a specific class based on the character of the concurrent vibro-acoustic emissions makes it possible to subsequently set such a mode of the drilling equipment which is evaluated by off-line experts as efficient mode for the given class. In the following figure are shown symptom vectors extracted from thirty realizations of the concurrent vibration signal from three types of rock. For reasons of possible visual display there are two-component symptom vectors $\mathrm{p}_i = (p_{i1}, p_{i2})$, where the symptom p_{i1} is calculated as the norm of the sampled signal (realization) with the length of $n = 1024$ samples:

$$p_{i1} = \|\mathrm{x}\| = \sqrt{\sum_{k=0}^{1023} x_k^2} \equiv L_2 x\,, \qquad (13)$$

and the symptom p_{i2} is the „peak-to-peak"parameter given by the relation:

$$p_{i2} = \left|\max(x_0, x_1, \ldots, x_{1023}) - \min(x_0, x_1, \ldots, x_{1023})\right| \qquad (14)$$

Fig.1 documents good differentiability of three rocks in Euclidean symptom space E_2.

In Table 1 are given the distances between the centroids of symptom vectors in the space E_9 with scalable symptoms into the interval $\langle 0,1 \rangle$ by using the metric (1). All 9 symptoms are extracted form the concurrent acoustic signal from the process of rotary drilling of five rocks. The results confirm once again the high differentiability of the rocks being drilled in the symptom space E_9.

Tab.1 Mutual distance of centroids of symptom vectors in the space E₉; the symptoms are extracted from the concurrent acoustic signal

	Quartz	**Marble**	**Chamotte**	**Brick 2**	**Limestone**
Quartz	0,00	0,40	0,41	0,29	0,60
Marble	0,40	0,00	0,81	0,12	0,21
Chamotte	0,41	0,81	0,00	0,69	0,99
Brick 2	0,29	0,12	0,69	0,00	0,31
Limestone	0,60	0,21	0,99	0,31	0,00

5 Comparison of geophysical signals in Hilbert space of type $\mathbf{L} \equiv \mathbf{H}$

In the following, an application of the Hilbert space is again illustrated in the field of efficient control of the process of rock separation by rotary drilling. The signal of concurrent vibrations is understood here as an element of the space $L_p \langle 0,T \rangle \equiv H$, while this real signal of the accelerometer satisfies the conditions of continuity and bounded ness in the interval $\langle 0,T \rangle$ and can be expressed in the form (5). In Fig.2a is shown the time waveform of the realization of this signal and in Fig. 2b is a simplified illustrative depiction of this realization of the signal as a point in infinite-dimensional Hilbert space whose coordinates are individual values of the signal in time $t, t: 0 \to T$. The signal as a vector here is simply depicted as a mass point of coordinates of the vector. In Fig. 2c is a simplified illustrative depiction of the signal realization as a point infinite-dimensional Hilbert space whose coordinates are individual values of the amplitude spectrum of the signal for angular frequencies $\omega, \omega: 0 \to \infty$.

Realizations of signals of concurrent vibrations from the process of drilling of three rocks were analyzed in (Leššo, 2009), (Leššo, 2010). Each realization of the signal was transformed with the FFT algorithm into the vector $X_{|\,.\,|} = \left(\left| \hat{F}(\mathbf{i}\omega), \omega: 0 \mapsto \infty \right| \right)$. Subsequently, norms of these vectors were computed from the relation:

$$\left\| X_{|\,.\,|} \right\| = \sqrt{\sum_{k=0}^{1023} \left| \hat{F}(\mathbf{i}\omega_k) \right|^2} \equiv L_2 X_{|\,.\,|} \equiv \mathrm{L2FT}. \qquad (15)$$

This is a numerical realization of the relation (9). In Fig. 3 is given the behaviour of this norm for thirty realizations of each of the three rocks. Since the norm of the vector represents its distance form the origin of the coordinate system, the difference in values suggests demonstrable differentiability of the analyzed rocks in Hilbert space.

Used the metric (7) applied to the vectors expressed respect to the basis of harmonic functions (frequency domain, vector $X = \left(\hat{F}(\mathbf{i}\omega), \omega: 0 \mapsto \infty \right) \in L_p \langle 0, \infty)$).

In Table 2 are given calculations of mutual distances of a triple of analyzed rocks in Hilbert space.

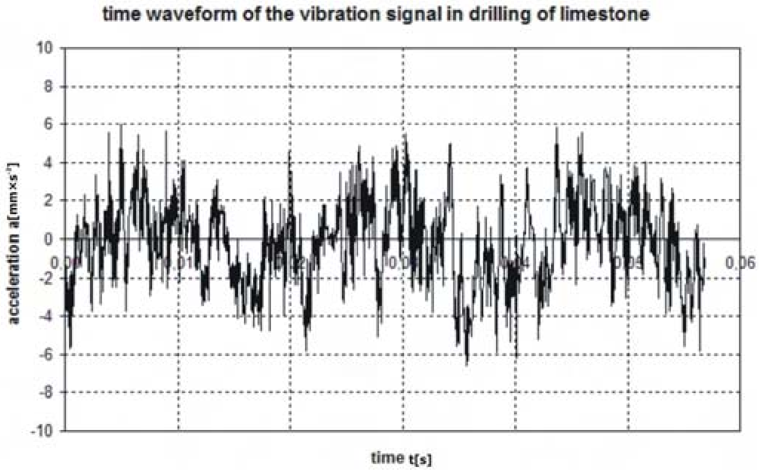

Fig.2a Time waveform of the vibration signal in drilling of limestone (one realization of the signal)

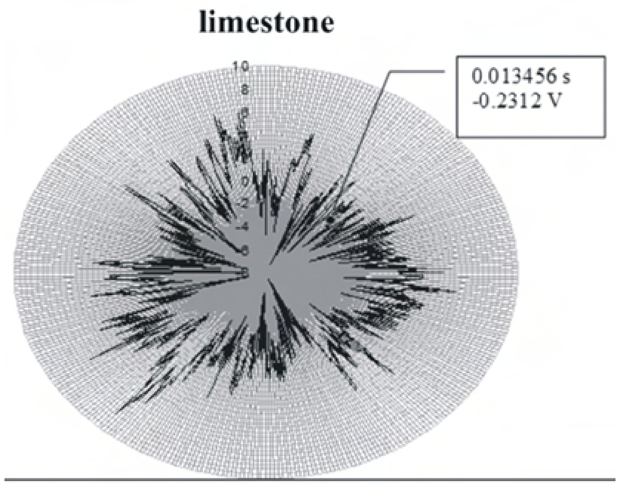

Fig.2b Simplified interpretation of the signal from Fig.2a as a vector (point) $x = (x(t), t{:}0 \longmapsto T)\ L_p < 0,T >$ in infinite-dimensional real Hilbert space $L_p < 0,T >$; time waveform of signal

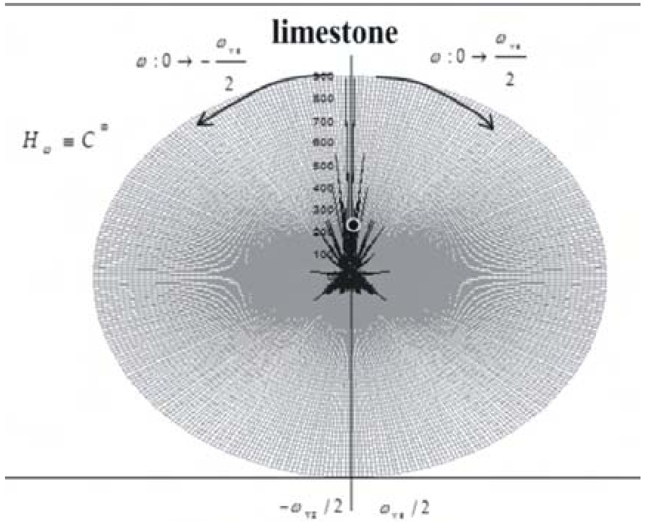

Fig.2c Simplified interpretation of the signal from Fig.2a as a vector (point) $X = (|\hat{F}(\mathrm{i}\omega)| , \omega{:}0 \longmapsto \infty)$ in infinite -dimensional Hilbert space $L_p < 0, \infty >$; amplitude frequency spectrum of the signal

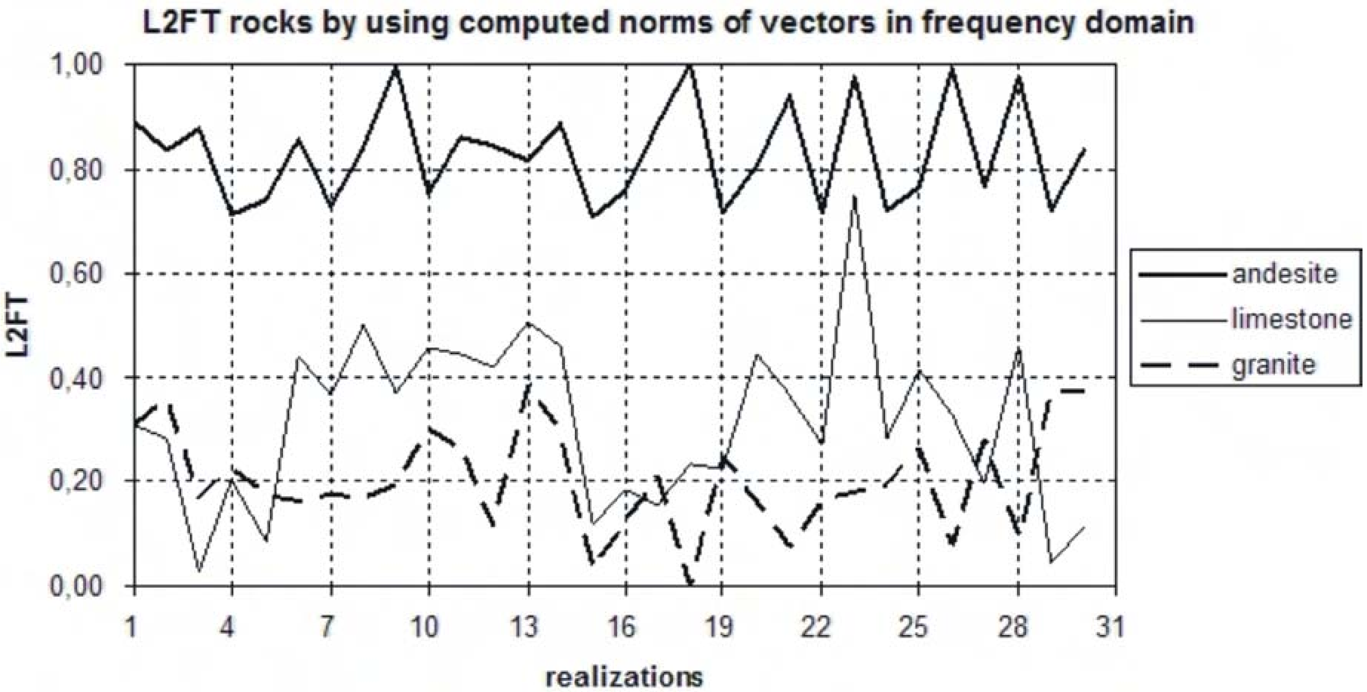

Fig.3 Differentiability of analyzed rocks in Hilbert space by using computed norms of vectors in frequency domain

Tab.2 Mutual distances of vectors of rocks in space $L_p \equiv H$ in frequency domain

Rock		$L_p \equiv H$
x_i	x_j	$\rho(x_i, x_j)$
Andesite	Granite	909
Limestone	Granite	469
Andesite	Limestone	912

In Fig.4 is a virtual depiction of three analyzed rocks in 3D space, when the calculated norms and mutual angles have been used. The figure is sort of a projection of rocks from infinite-dimensional Hilbert space into 3D Euclid space. Demonstrated is good differentiability of rocks on the basis of concurrent vibrations in rotary separation.

6 Conclusions

In the contribution are given partial results of scientific research of a group of researchers who investigate the possibility of using mathematical theory of abstract spaces in the field of processing of geophysical signals. The core of the research at present is an application of these methods in areas of control of the process of rock separation by rotary drilling.

First experiments also concern processing of seismic signals in engineering seismology. The authors present basic theoretical foundations of the subject of metric spaces of Euclidean and Hilbert type, while preferring better

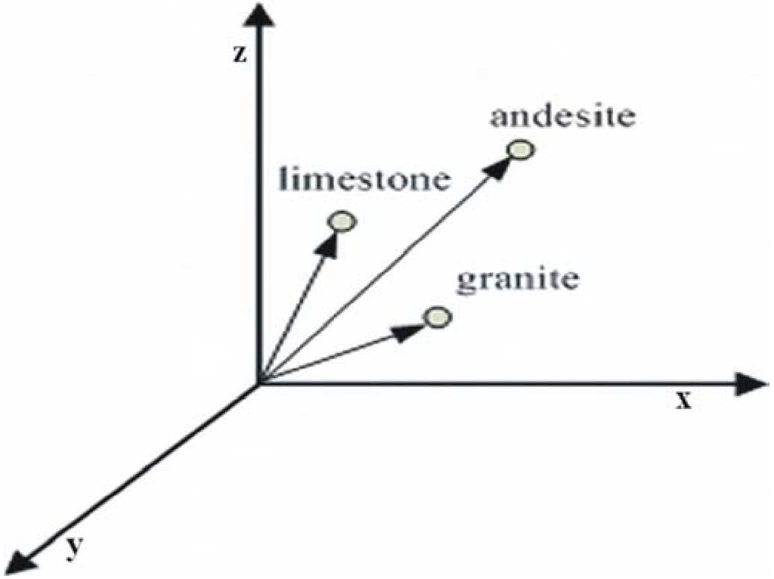

Fig.4 Virtual locations of analyzed rocks in 3D space corresponding to average values of norms and values of mutual angles of rocks as vectors in Hilbert space

lucidity before mathematical rigor. Figures and tables illustrate relatively good capability of the used methods and algorithms to differentiate geophysical signals based on their properties. The ultimate goal of the research oriented this way is to use abstract Hilbert spaces for the classification and recognition of geophysical signals while taking into account previously defined templates of individual classes of signals. For the creation of class templates can be used, for example, the algorithm of vector quantization. The techniques and methods given in the paper can be connected with the methods of artificial intelligence. In further research attention will be paid to utilization of these methods in seismic tomography, where the influence of the inner structure of rock block on the transmission of artificially generated seismic signals, and associated possibilities of visualization, will be investigated.

Declaration

This article was written thanks to the support within OP Research and development for the project: ITMS 26220220031, co-funded from European fund of regional development. The article was written also under support from projects MŠ VEGA 1/2578/12.

References

LEŠŠO, I. & FLEGNER, P. [et al.] VEGA 1/4194/07 *Výskum exaktných metód hodnotenia efektívnosti a kvality procesu rozpojovania hornín rotačným vŕtaním*, Záverečná správa, FBERG TU v Košiciach, 2009.

NAYLOR, A.W. & SELL, G.R. *Teória lineárnych operátorov v technických a prírodných vedách*, Alfa 1981, 1. vydanie. ISBN: 63-175-81.

TAYLOR, A.E. *Úvod do funkcionální analýzy*, Praha, 1973, Academia.

KREPELKA, F. & CHLEBOVÁ, Z. & IVANIČOVÁ, L. *Meranie, analýza a vyhodnocovanie prevádzkových náhodných procesov pri vŕtaní hornín*. In Acta Mechanica Slovaka, optimalizácia mechanických sústav. ISSN 1335-2393, 2008, roč. 12. č. 3-C, p. 229-136.

PANDULA, B. & KONDELA, J. *Metodológia seizmiky trhacích prác*, Slovenská spoločnosť pre trhacie a vŕtacie práce – ZSVTS, Bamská Bystrica, Košice, 2010.

LEŠŠO, I. & FLEGNER, P. [et al.] *Application of vector quantisation method for effective and quality process control of rock disintegration by rotary drilling*. In SGEM 2009. Volume 2. - Sofia: Bulgarian academy of sciences, 2009 p. 115-122, ISBN 9549181812

LEŠŠO, I. & FLEGNER, P. [et al.] *Research of the possibility of application of vector quantisation method for effective process control of rocks disintegration by rotary drilling*. In Metalurgija. Vol. 49, no. 1 (2010), p. 61-65, ISSN 0543-5846 Spôsob prístupu:
http://public.carnet.hr/metalurg/Metalurgija/2010_vol_49/No_1/MET_49_1.pdf.

Authors

[1] prof. Ing. Igor Leššo, CSc., Ústav riadenia a informatizácie výrobných procesov, Fakulta BERG, Technická univerzita v Košiciach, Letná 9, 040 01 Košice, Slovenská republika, tel.: (+421) 055 6025178, e-mail: igor.lesso@tuke.sk.

[2] Ing. Patrik Flegner, Ph.D., Ústav riadenia a informatizácie výrobných procesov, Fakulta BERG, Technická univerzita v Košiciach, Letná 9, 040 01 Košice, Slovenská republika, tel.: (+421) 055 6025174, e-mail: patrik.flegner@tuke.sk.

[3] Ing. Katarina Feriančíková, Ústav riadenia a informatizácie výrobných procesov, Fakulta BERG, Technická univerzita v Košiciach, Letná 9, 040 01 Košice, Slovenská republika, tel.: (+421) 055 6025190, e-mail: katarina.feriancikova@tuke.sk