VISUALIZATION OF VIBRO - SIGNALS IN HILBERT SPACES

VIZUALIZÁCIA VIBRO - SIGNÁLOV V HILBERTOVÝCH PRIESTOROCH

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Abstract

Hilbert spaces are a higher, mathematical abstraction of Euclidean spaces of linear algebra. They represent a key notion in functional analysis as a chapter in modern mathematics. The elements of the Euclidean space, which is finite-dimensional, are linear algebraic vectors as ordered *n*-ads of real numbers. The elements of a Hilbert space, which is infinite-dimensional complex space, are mathematical functions satisfying certain conditions. The set, topological, algebraic, and geometrical structure of these spaces enables to analyze mutual relations between functions as vectors. It is possible to generalize and subsequently to express numerically geometric notions such as e.g. size of function, angle subtended by two functions, distance of functions, etc. The authors present the use of Hilbert spaces in the solution of geophysics tasks when they solve the problem of how, in 3-D space, to graphically visualize the mutual geometric location between geophysics signals viewed as points – vectors of infinite-dimensional Hilbert space.

Abstrakt

Hilbertove priestory sú vyššou matematickou abstrakciou Euklidových priestorov lineárnej algebry. Predstavujú kľúčový pojem vo funkcionálnej analýze ako kapitole modernej matematiky. Prvkami euklidovského priestoru, ktorý je konečne rozmerný, sú lineárne algebraické vektory ako usporiadané *n*–tice reálnych čísel. Prvkami Hilbertovho priestoru, ktorý je nekonečne rozmerný komplexný priestor, sú matematické funkcie, spĺňajúce určité vlastnosti. Množinová, topologická, algebraická a geometrická štruktúra týchto priestorov umožňuje analyzovať vzájomné vzťahy medzi funkciami ako vektormi. Je možné abstrahovať a následne číselne vyjadriť také geometrické pojmy ako napr. veľkosť funkcie, uhol zvieraný dvoma funkciami, vzájomná vzdialenosť medzi funkciami, a pod. Autori prezentujú využitie Hilbertových priestorov pri riešení geofyzikálnej úlohy, pričom v príspevku riešia problém ako v priestore 3D

graficky zviditeľniť vzájomnú geometrickú polohu medzi geofyzikálnymi signálmi ako bodmi – vektormi nekonečne rozmerného komplexného Hilbertovho priestoru.

Keywords

quality process, metric space, Hilbert space, geophysics signal, classification, vector quantization of space

Kľúčové slová

proces kvality, metrický priestor, Hilbertov priestor, geofyzikálny signál, klasifikácia, vektor kvantovania priestoru

1 Introduction

The problem of control of the rock separation process has its specifics. The main problem is the fact that this process is intrinsically involved and its state quantities are non-measurable with standard methods in real-life conditions. The key question is the sufficiency of information about the effect of the drilling mode on the very behaviour of separation of particular rock (Leššo et al., 2010), (Leššo et al., 2009). Under the term „mode of separation"we understand the synergic effect of main technological components of the process of drilling which are the following action quantities: pressure of the drilling tool on the front of the drill F [N], revolutions of the drilling tool n [revs^{-1}], flow capacity of the lavage during time unit Q [m^{3}s^{-1}] and the quality of the lavage, given by its physicochemical parameters. All these components are independent of each other; it is possible to control them individually during drilling. The knowledge of the drilling modes is the foundation of the knowledge of the process of rock separation by rotary drilling.

From the system point of view the process of rock drilling can be understood in simplified form as the system characterized by the set of quantities some of which we can affect, but some not Fig.1.

Both parameters w and φ are the state quantities of the process, which are not measurable directly under real-life conditions. More state quantities affect the process of separation: properties of the indentor (drilling tool) and the geomechanical properties of the rock massif being separated.

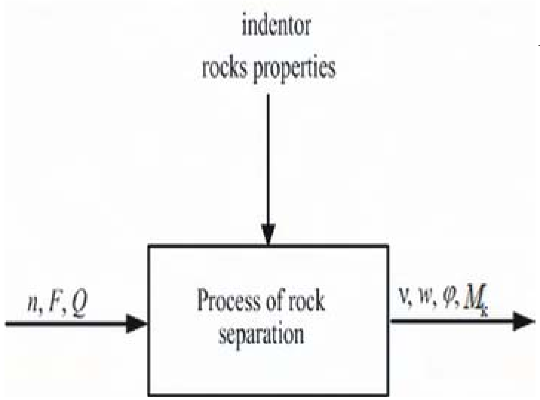

Fig. 1 Process of rock separation as the object of control, controlled system n revolutions of the drilling tool [rpm], F - pressure force [N], Q - volume of lavage of the drill with water [$m^3 \times s^{-1}$], v - speed of drilling [$mm \times s^{-1}$], M_k - torque [Nm], w - specific work of separation [$J \times m^{-1}$], φ = υ/w - work capability of the tool [$m^4 \times J^{-1} \times s^{-1}$].

The task of synthesis of control of the process of drilling the rock massif, defined in this way, with the requirement to maintain efficient mode under the condition of maximization of the objective function, encounters the problem that it is unrealistic to create adequate analytical mathematical model of the process of rock separation by rotary drilling. This is because this process is too involved from the viewpoint of fundamental modelling and empirically obtained models are only valid under specific conditions. However, the monitored process of separation is strongly stochastic and non-stationary under the influence of changing geomechanical conditions and also under the influence of the indentor wear. Added to this problem are problems with measurability of the objective function under real conditions (Leššo et al., 2008), (Leššo et al., 2007), (Panda, 2000), (Panda, 2010), (Pinka et al., 2010).

The solution is to avoid the classical system of control using a model of the process and to solve the system of control based on some of the modern methods of control of complex processes where the source of complexity can be nonlinearity, instability, difficult to describe stochasticity, non-measurability of parameters, multiplicity etc. Among such modern methods of control belong also the methods based on so-called artificial intelligence.

In research of the question of efficient control of the process of rock separation by rotary drilling the basis was the intuitive idea that the accompanying vibro-acoustic signal contains information about the character of the process of separation from the viewpoint of geomechanical properties of the rock and from the viewpoint of efficiency of the process of drilling itself. This idea was verified experimentally.

2 Method descriptions

Hilbert spaces are infinite-dimensional spaces in which functions usually represent the points – vectors. The coordinates of such vectors are then the function values of given function in individual points of the definition domain. Each function value is in general a complex number. For the purposes of utilization of Hilbert spaces in the solution of specific tasks, e.g. tasks in the area of digital signal processing, the geometric structure of these spaces is significant and it provides an interesting tool for investigation of mutual relations between functions – signals as vectors of the space (Naylor, 1981), (Taylor, 1973), (Hansen, 2006). These mutual relations can be expressed quantitatively by using exact analytical expressions based on linear algebra. It is mainly about the norms of functions, angles between functions, angles between a function and chosen axis of the coordinate system and also about the mutual distances between functions according to the metric defined. In Fig.2 is shown the situation when there is a couple of functions f_i, f_j and denoted are three numerical characteristics expressing their mutual geometric relations: norm of the function, metric distance between functions and their mutual angle. The triple of such indicators enabled to illustrate the situation by using 3-D space.

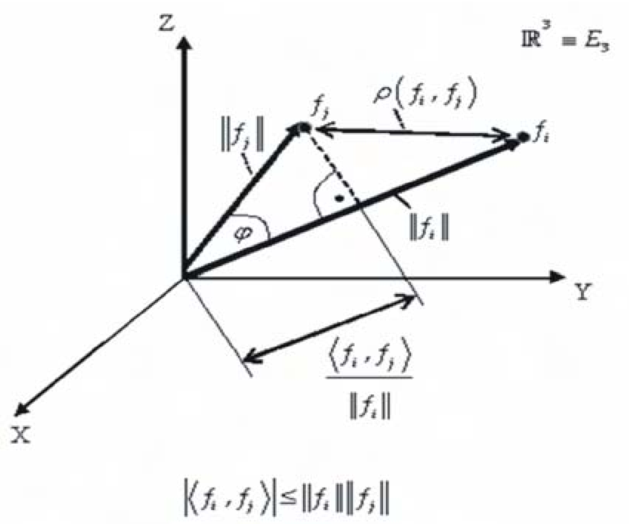

***Fig. 2** Illustrative graphical depiction of mutual geometric relations of functions as points of Hilbert space. Geometric relations follow from the Schwarz–Cauchy–Bunyakowsky inequality*

On the other hand, practice shows that evaluation of spatial relations among several functions as points of Hilbert space based on numerically expressed parameters is involved and cannot substitute empirical perception of space analogously as in 3-D space. Therefore each method enabling visualization of mutual location of functions in Hilbert space by means of 2-D or 3-D Euclidean space can be very useful in practice. In this contribution is shown one of the possible solutions of this problem.

The notion „function“in mathematics has several forms of interpretation. In functional analysis to each of these forms there corresponds a certain class of Hilbert's spaces. Physical signal read by a sensor can be understood as a bounded

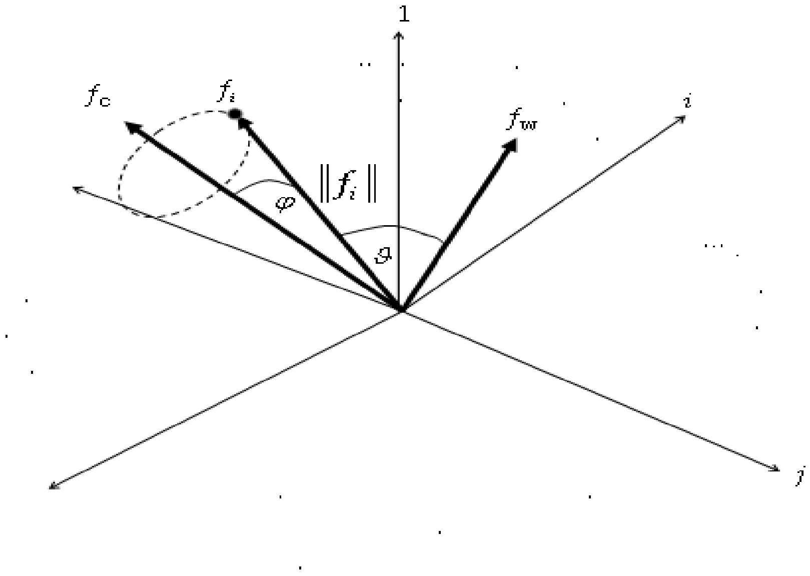

Fig. 3 Method of unique determination location of function in the infinite dimensional Hilbert space; using of two reference functions

real or complex function of real variable, defined over a time interval $t \in \langle a,b \rangle$.

From a mathematical viewpoint, such a function can be understood as an infinite sequence of function values, i.e. $f = \left(f(t), t : a \mapsto b\right) = \left(f_1, f_2, ..., f_k, ...\right)$. Hilbert spaces whose elements are such functions make up a class of spaces $l_{p\langle a,b \rangle}$. A Hilbert space from this class is then a set of all infinite sequences of real or complex numbers such that the series of members of the sequence converges.

Consider Hilbert space H of class $l_{p\langle a,b \rangle}$. Its elements are functions $f_i = \left(f_i(t); t : a \mapsto b\right) \in l_{p\langle a,b \rangle}$, i=1, 2 ... The task of visualization of location of functions in Hilbert space by means of 3-D space can be defined from a mathematical viewpoint as a task of finding a one-to-one map $F: H \to E_3$, which assigns one and only one vector $y_i \in E_3$ for any function $f_i \in H$. Then for each pair $f_i \neq f_j$ from the space H holds $F(f_i) \neq F(f_j)$.

In finding such a map *F*, one can use the fact that a vector in space $E_3 \equiv \mathrm{R}^3$ is uniquely identified by a triple of real numbers; see the body R. From this it follows the necessity to define in the space H a triple of such real numerical parameters that would together identify the location of function as vector in space H. In Fig.3 is illustrated the situation when in Hilbert space function f is a vector.

For the purposes of unique identification of the location of function f_i in Hilbert space are defined two reference functions – vectors, namely white noise f_W and centroid f_C of all functions from this space. In Fig.3 it is shown that the size of angle φ, which is subtended by function f and centroid f_{C} along with the norm $\|f\|$ is not unique information about the location of the function since there is are an infinite number of such functions and their geometric location is the depicted circle. This ambiguity follows also from the Schwarz – Cauchy inequality:

$$\cos\varphi = \frac{\left|\left(f, f_{\mathrm{C}}\right)\right|}{\|f\|\|f_{\mathrm{C}}\|} = \frac{\left|\sum_{k=0}^{\infty} f_k f_{\mathrm{C}k}\right|}{\sqrt{\sum_{k=0}^{\infty} f_k^2}\sqrt{\sum_{k=0}^{\infty} f_{\mathrm{C}k}^2}}. \tag{1}$$

After modification we obtain the equation whose structure shows it is the equation of the circle in Hilbert space:

$$\|f\|\|f_{\mathrm{C}}\|\cos\varphi = \left|\sum_{k=0}^{\infty} f_k f_{\mathrm{C}k}\right|. \tag{2}$$

It follows from the above ambiguity of identifying function location by only one angle that it is necessary to simultaneously consider angle subtended by given function f with another reference vector. Specifically, as a second vector there was defined the vector of white noise f_{W}.

So the proposed method of identifying the location of function f in Hilbert space uses two reference vectors:

- White noise vector

$$f_{\mathrm{W}} = \left(f_{\mathrm{W}}(t); t : a \mapsto b\right) \in l_p. \tag{3}$$

It synthesizes the harmonic components of all frequencies, so that

$$f_{\mathrm{W}}(t) = \frac{1}{|a-b|}\int_{-\infty}^{\infty} \hat{F}_{\mathrm{W}}(\mathbf{i}\omega)\mathrm{e}^{\mathbf{i}\omega t}\,\mathrm{d}\omega. \tag{4}$$

- Centroid f_{C} of all functions located in this space:

$$f_{\mathrm{C}} = \left(f_{\mathrm{C}}(t); t : a \mapsto b \right) \in l_{p\langle a,b\rangle}, \tag{5}$$

where

$$f_{\mathrm{C}}(t) = \frac{1}{N} \sum_{i=1}^{N} f_i(t), \text{ for } t : a \mapsto b. \tag{6}$$

To provide good sensitivity and accuracy in the calculation of mutual geometric relations between signals it is suitable for the white noise as a synthetic signal to be defined so that its energy was identical to that of the centroid. For the energy of the centroid we have:

$$E_{\mathrm{C}} = \left\| f_{\mathrm{C}} \right\|^2 = \int_a^b \left| f_{\mathrm{C}}(t) \right|^2 \mathrm{d}t. \tag{7}$$

Analogously for the energy of white noise:

$$E_{\mathrm{W}} = \left\| f_{\mathrm{W}} \right\|^2 = \int_a^b \left| f_{\mathrm{W}}(t) \right|^2 \mathrm{d}t. \tag{8}$$

For the purposes of synthesis of white noise as a reference signal with required energy it is suitable to express this noise in the form of Fourier series as a sum of harmonic components of all frequencies with the same amplitude, defined in the interval $\langle a,b \rangle$ in the space $l_{p\langle a,b\rangle}$:

$$f_{\mathrm{W}}(t) = \sum_{k=-\infty}^{\infty} F_{\mathrm{W}}(\mathrm{i}\omega_k) \mathrm{e}^{\mathrm{i}\omega_k t}, \; F_{\mathrm{W}}(\mathrm{i}\omega_k) \in \mathrm{C}, \; F_{\mathrm{W}}(\mathrm{i}\omega_k) = \mathrm{const.}. \tag{9}$$

By substituting (9) into (8) we obtain for the white noise with the energy equal to the energy of centroid the relation:

$$E_W = \int_a^b |f_W(t)|^2 \, dt = \int_a^b \left(\sum_{k=-\infty}^{\infty} |F_W(\mathbf{i}\omega_k)| \right)^2 dt = \left(\sum_{k=-\infty}^{\infty} |F_W(\mathbf{i}\omega_k)| \right)^2 |b-a| . \quad (10)$$

For the energy of white noise in the relation (10) to be finite it is necessary that its spectrum be frequency bounded. If we bound it with the Nyquist frequency in the interval $\langle -\omega_{vz}/2, +\omega_{vz}/2 \rangle$, we get, while taking into account (9):

$$E_W = \left(B |F_W(\mathbf{i}\omega_k)| \right)^2 |b-a| = E_C . \quad (11)$$

From expression (11) we then obtain the relation for the magnitude of the amplitude which is the same for all of its components in the band B:

$$\left| \hat{F}_W(\mathbf{i}\omega_k) \right| = \frac{1}{B} \sqrt{\frac{E_C}{|b-a|}}, \; k \in B . \quad (12)$$

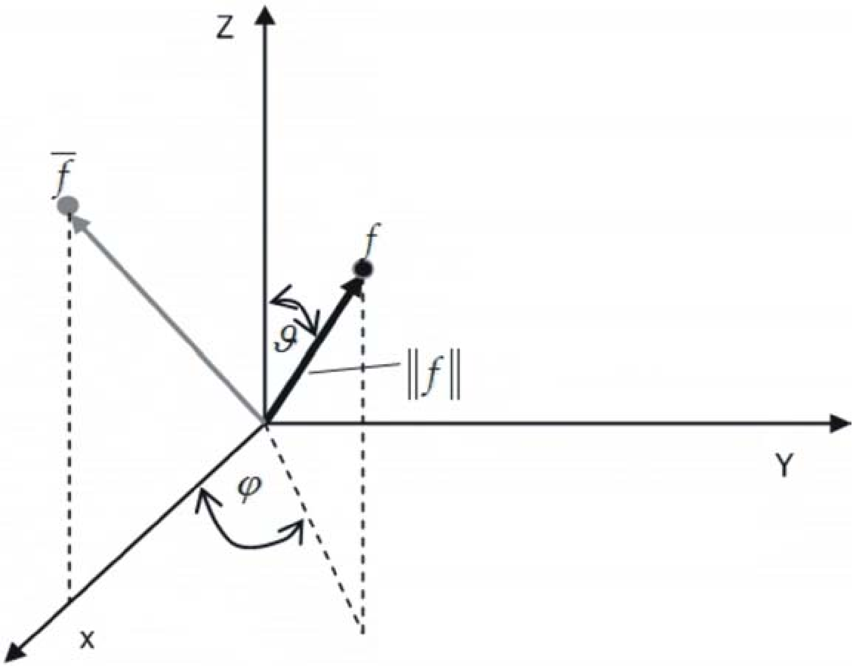

Fig. 4 Mapping of Hilbert space function in space by using transformation of polar coordinates into cartesian coordinates.

The mentioned three parameters, angle with centroid $\varphi \equiv \sphericalangle (f, f_c)$, angle with white noise $\upsilon \equiv \sphericalangle (f, f_w)$ and the norm of the vector $\|f\|$ can be used as coordinates of the vector in space E_3 while using spherical coordinates, see Fig.3. These spherical coordinates can be transformed to Cartesian coordinates with the known transformation formulae, which lead to a point in the Cartesian coordinate system as a three of real numbers.

$$\left(\|f\| \sin(\vartheta) \cos(\varphi), \|f\| \sin(\vartheta) \sin(\varphi), \|f\| \cos(\vartheta) \right). \quad (13)$$

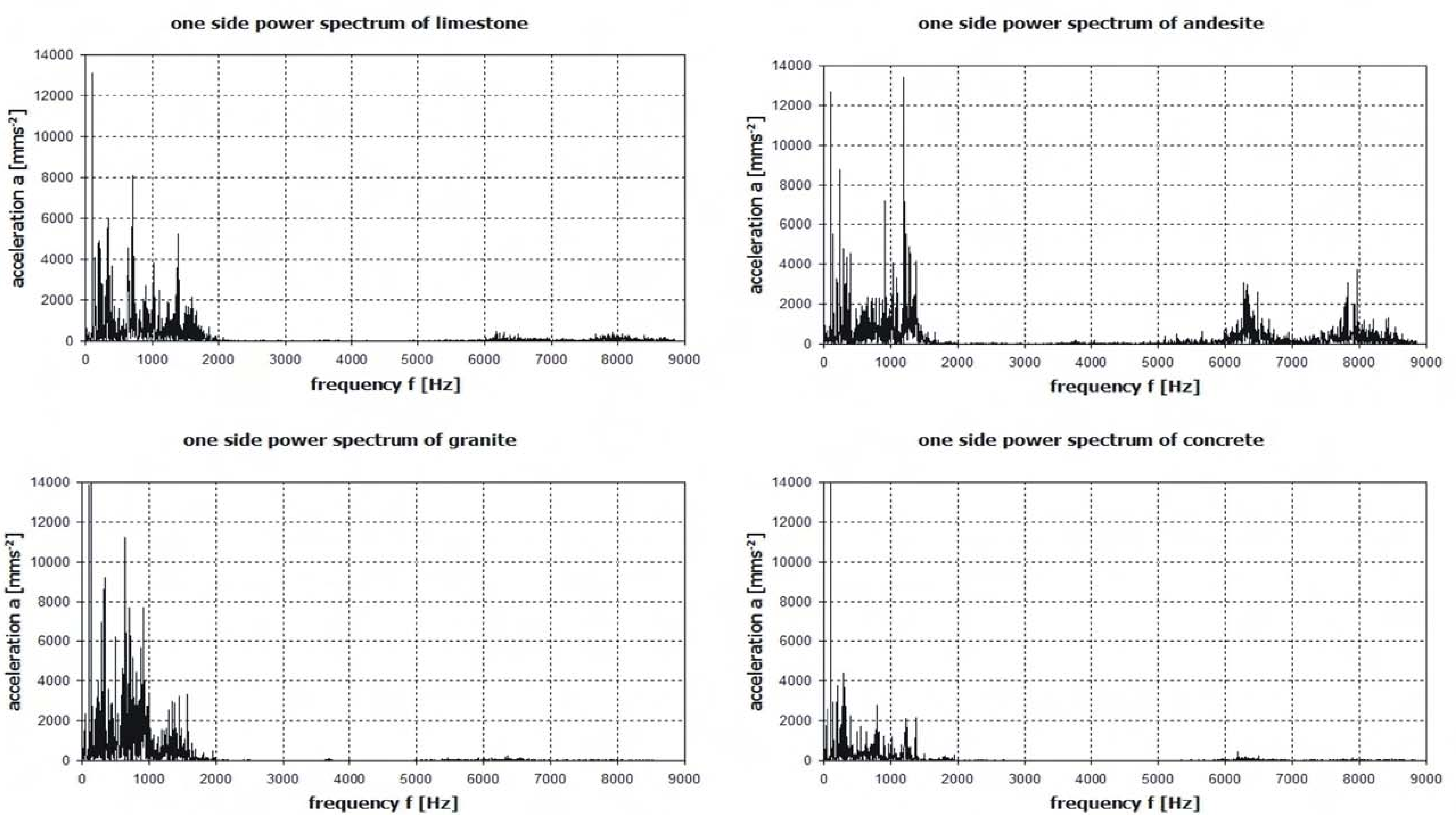

Fig. 5 One side power spectrum of rocks limestone, andesite, granite, concrete.

It follows from the structure of the relations used for the calculation of the angles that this way the functions in Hilbert space are mapped into one quadrant of the space E_3 only. The above method of mapping the vector in Hilbert space into the space E_3 is illustrated in Fig.4.

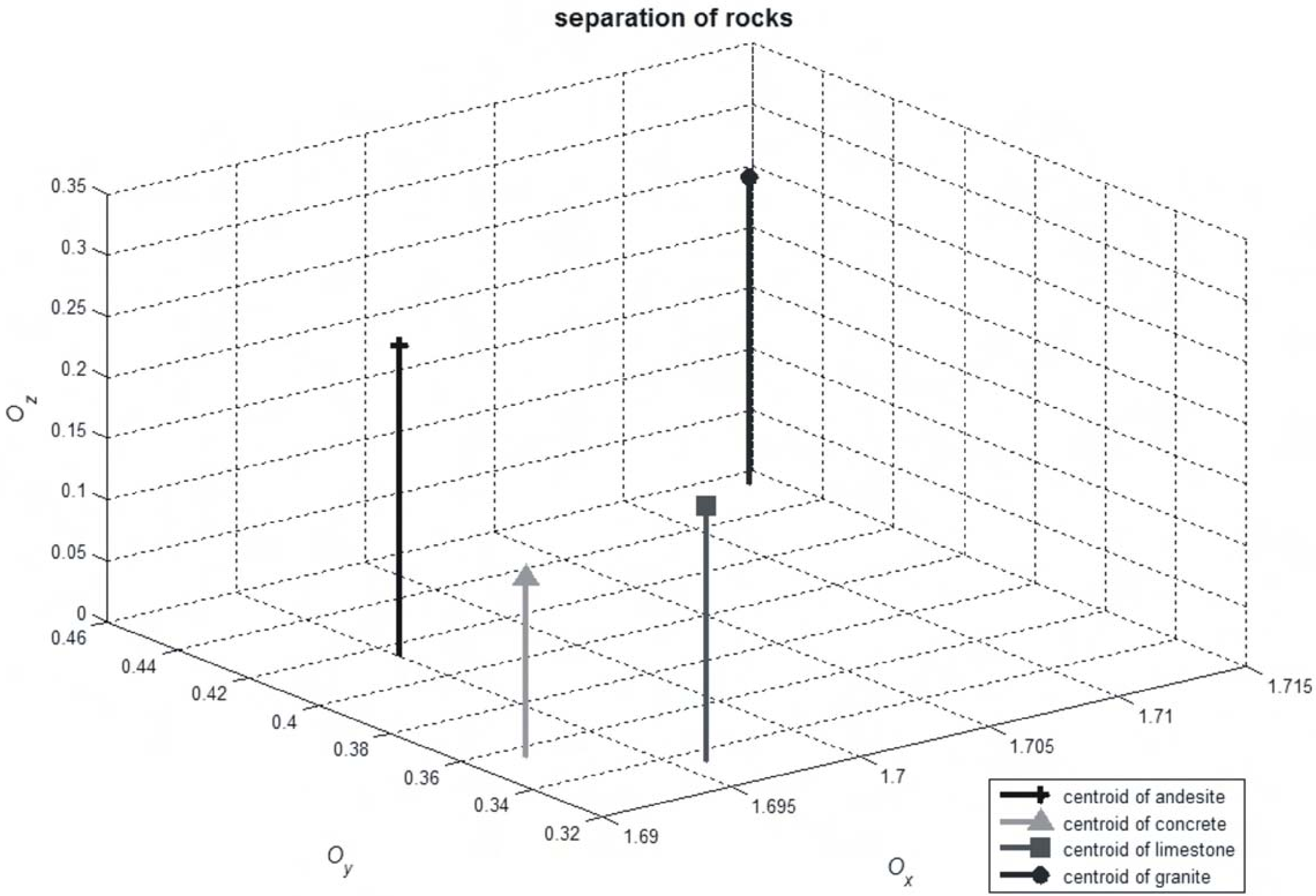

Fig. 6 Visualization in 3D of differentiability of four separated rocks represented by concurrent vibrations as vectors in Hilbert space, O_x - axis x, O_y - axis y, O_z - axis z, correspond with the expression (13)

3 Application of method in effective control of the process of rotary drilling

The above method was experimentally used in the solution of the task of effective control of the process of rotary drilling of rock massif (Krepelka et al., 2008), (Leššo et al., 2012), (Leššo et al., 2011). The essence of the proposed control algorithm is classification of the rock being separated on the basis of concurrent vibration signal. Recognition of the class of the rock based on the concurrent vibro-acoustic emissions, in which also its geomechanical properties are manifested, enables to subsequently choose form the table of experts the corresponding mode of drilling which is efficient for given class of rock. The mode will provide minimal specific energy of separation at maximal speed of drilling. From realizations of concurrent vibrations of the process of separation of four types of rock (A – andesite, V – limestone, Z – granite, B – concrete) were calculated the power spectra. They represent, in the sense of above considerations, the functions – vectors of Hilbert space Fig.6. In view of random, but stationary character, these power spectra were averaged for each rock from 30 realizations of the signal Fig.5. The mode of drilling (pressure, revolutions) was stabilized during the measurement. On the basis of the above procedure, calculation of the three of digital positional characteristics was performed for the centroid of each rock. Those characteristics then determined a unique location in 3-D space Fig.6 for the centroid of each rock.

The above method of visualization was used also for showing the fact that the concurrent vibrations contain information not only about the class of the rock being separated, from the viewpoint of its geomechanical properties, but also information about the current mode of drilling. In Fig.7 is shown the trajectory of the movement of power spectrum of concurrent vibrations as a vector of Hilbert space, namely at gradual increase of pressure F= 10000 [N].

4 Conclusions

The above methods of processing physical signals using the so-called new mathematics enable solution of many complex problems. Functional analysis, namely Hilbert spaces, makes it possible to analyze signals as functions and generalize geometric relations among them. Problematic, however, is the fact that these geometric relations of signals as vectors in Hilbert space cannot be empirically perceived as in the case of vectors in Euclidean 3D space. This contribution tries to show one of the possible solutions of this problem.

The proposed method of unique map of vectors of Hilbert space into the 3-D space was applied on geophysical vibration signals from the process of rotary drilling of rock (Pandula et al., 2010). Visualized was the differentiability

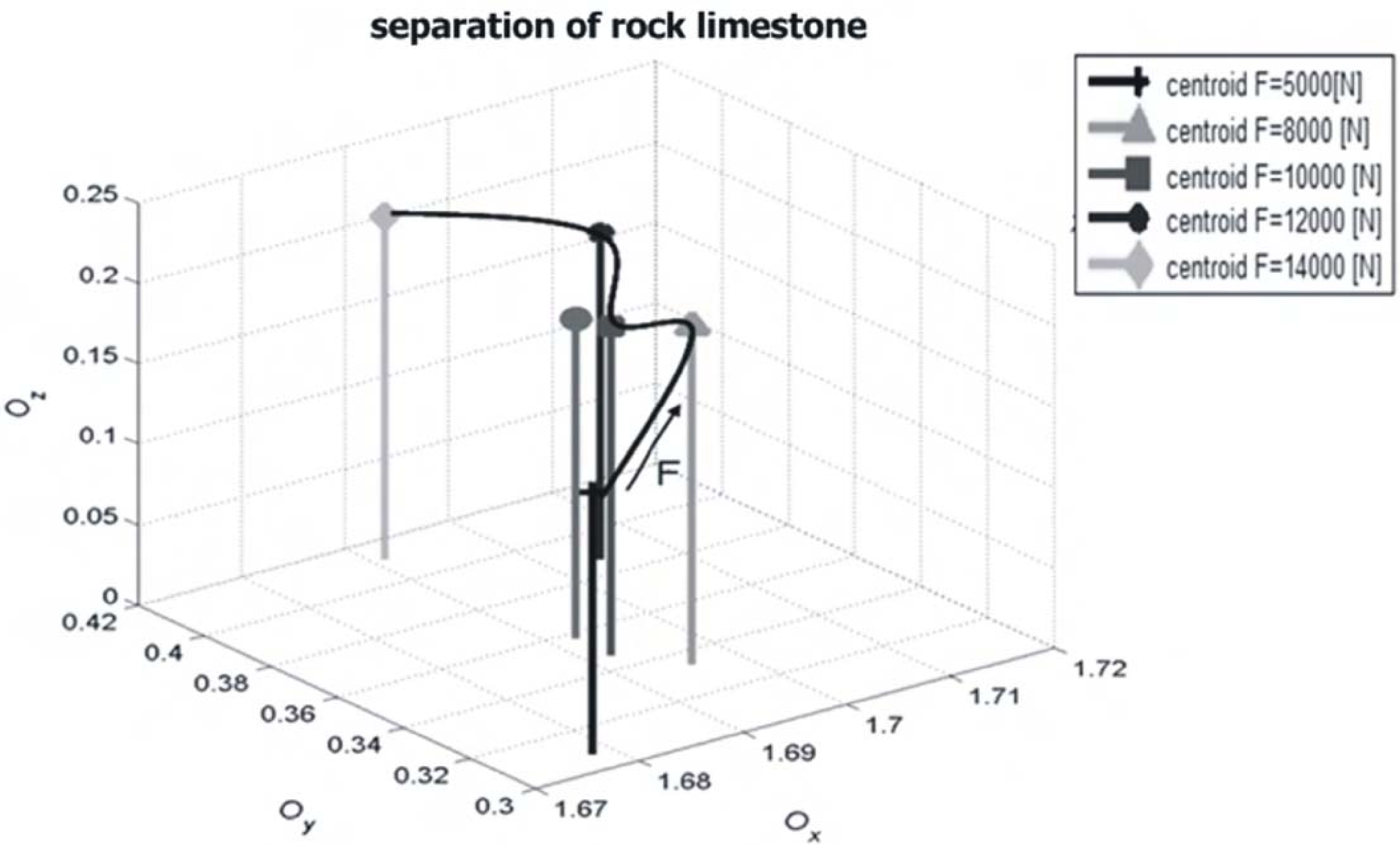

Fig. 7 Trajectory of concurrent vibrations of the process of drilling limestone at gradual increase of pressure F [N].

of four kinds of rock on the basis of vibrations and also the sensitivity of the location of vibration signal in Hilbert space on the mode of drilling. This method does not require a classical analysis of selected parts of the spectrum, using spectrum over the frequency range. This full spectrum represents of drilled rock as a vector in status Hilbert space.

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References

HANSEN V. L. Functional analysis – Entering Hilbert space. *In World Scientific,* Singapore, 2006, ISBN 981–256–563–9, ISBN 981–256–686–4

KREPELKA, F. & CHLEBOVÁ, Z. & IVANIČOVÁ, L. Meranie, analýza a vyhodnocovanie prevádzkových náhodných procesov pri vŕtaní hornín. *In Acta Mechanica Slovaka, optimalizácia mechanických sústav*. ISSN 1335–2393, 2008, roč. 12. č. 3–C, p. 229–136.

LEŠŠO I. & FLEGNER P. & FERIANČIKOVÁ K. Hilbert spaces as a specific instrument for process control. *In ICCC 2012 proceedings of the 13th International Carpathian Control Conference*, Podbanské, Slovak Republic, May 28–31, 2012, Košice TU, 2012 pp. 430–435, ISBN 978–1–4577–1866–3

LEŠŠO I. & FLEGNER P. & LACIAK M. & FERIANČIKOVÁ K. Hilbert spaces as an efficient instrument for signal processing in geotechnics application of vector quantisation method in Hilbert space for effective and quality process control of rock disintegration by rotary drilling. *In ICCC 2011 proceedings of the 12th International Carpatian Control Conference*, 25–28 May 2011, Velké Karlovice, Czech Republic, 2011 pp. 253–256, ISBN 978–1–61284–359–9

LEŠŠO, I. & FLEGNER, P. [et al.] Research of the possibility of application of vector quantisation method for effective process control of rocks disintegration by rotary drilling. *In Metalurgija. Vol. 49, no. 1* (2010), pp. 61–65, ISSN 0543–5846 Spôsob prístupu: http://public.carnet.hr/metalurg/Metalurgija/2010_vol_49/No_1/MET_49_1.pdf.

LEŠŠO I. & FLEGNER P. [et al.] Application of vector quantisation method foe effective and quality process control of rock disintegration by rotary drilling. *In SGEM 2009*, Volume 2, Sofia, Bulgarian academy of sciences, 2009, pp. 115–122, ISBN 9549181812

LEŠŠO, I. & FLEGNER, P. [et al.] VEGA 1/4194/07 *Výskum exaktných metód hodnotenia efektívnosti a kvality procesu rozpojovania hornín rotačným vŕtaním*, Záverečná správa, FBERG TU v Košiciach, 2009.

LEŠŠO I. & HOROVČÁK P. & FLEGNER, P & PANDULA B. Process control by rotary drilling with exploation of artificial intelligence methods. *In SGEM 2008,* Volume 2, Sofia SGEM, 2008 pp. 505–512, ISBN 9549181812

LEŠŠO I. & FLEGNER P. & PANDULA B. & HOROVČÁK P. New principles of process control in geotechnics by acoustic methods. *In Metalurgija,* Vol. 46, no. 3 (2007), pp. 165–168, ISSN 0543–5846

NAYLOR, A.W. & SELL, G.R. *Teória lineárnych operátorov v technických a prírodných vedách*, Alfa 1981, 1. vydanie. ISBN: 63–175–81.

PANDULA, B. & KONDELA, J. *Metodológia seizmiky trhacích prác*, Slovenská spoločnosť pre trhacie a vŕtacie práce – ZSVTS, Banská Bystrica, Košice, 2010.

PANDA A. & PANDOVÁ I. Statistical Process Control, Principles of Statistics. *Transactions of the Universities of Košice 2/2000*, TU Košice 2000, pp. 20–23, ISSN 1335–2334

PANDA A. Bearings part production, Steel cage. *IBEN Studia I materialy*, t. XXVI-XXIX, Nr. 1–2, Monografie, Institute for scienctific research and expertises, Gorzów, Poland, ISSN 0860–7761, IBEN Gorzow Wlkp, 2010

PINKA J. & WITTENBERGER G. Equipment and technology for the core drilling in Slovakia. *In The national magazine of natural gas*. Vol. 10 (2009–2010), pp. 46–62, ISSN 1454–2692

TAYLOR, A.E. *Úvod do funkcionální analýzy*, Praha, 1973, Academia.

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