INTERPRETATION OF THE TIME SPECTRUM OF THE INDUCED POLARIZATION

INTERPRETACE ČASOVÉHO SPEKTRA VYNUCENÉ POLARIZACE

František Ryšavý [1]

Abstract

This paper presents the procedure of separation characteristics of the time spectrum for induced polarization. Interpretation of the time spectrum is based on premise that beds can be either the one-component material or the more-component material. The first group – there are those beds having no invasion or having very deep invasion there. The second group – it is the beds having small or middle invasion, D_i /d < 8. The curvilinear relationship of exponential shape is disassembled on two exponential relations for invasion zone and non-invaded bed. We have two characteristics presenting two values of chargeability on condition that t = 0. The next two characteristics present the relaxation time for invasion zone and non-invaded bed. The last characteristic is the integral chargeability.

Abstrakt

Tato práce představuje proces separace charakteristik časového spektra vynucené polarizace. Interpretace časového spektra je založena na předpokladu, že vrstvy mohou být považovány buď za jednosložkové, nebo vícesložkové. Do první skupiny patří ty vrstvy, které nemají buď žádnou filtraci, nebo mají filtraci velmi hlubokou. Druhou skupinu tvoří vrstvy, které mají malou nebo střední filtraci s Di/d < 8. Nelineární závislost, která má exponenciální průběh, se skládá ze dvou exponenciálních relací pro zónu filtrace a zónu nepostiženou filtrací. Máme tak dvě charakteristiky, které prezentují dvě hodnoty dielektrické konstanty za podmínky, že t = 0. Další dvě charakteristiky představují relaxační čas pro zónu filtrace a zónu nepostiženou filtrací. Poslední charakteristikou je integrální dielektrická konstanta.

Keywords

time spectrum, integral characteristic, chargeability, the relaxation time, time window, time base, well-logging

Klíčová slova

časové spektrum, integrální charakteristika, polarizovatelnost, doba relaxace, časové okno, časová báze, karotáž

1 Introduction

We can register in the borehole there where the slowly changing polarization field with time for every depth point is. Reaction on the current pulse takes long time; there are approximately tens or even hundreds of milliseconds. If I want to register the time spectrum simultaneously the time base remarked as t_b must be large enough; otherwise, it has such consequence that an influence of mud and invasion zone will dominate there and contribution of the non-invasion zone will be negligible.

Earlier when was produced time analysers having short time base there were often made the point registrations instead of the simultaneous registration. Nevertheless, this way had not been enlarged in recent years, because the recent time analysers present very sophistic instrument having time base long enough to be possible to register induced polarisation simultaneously in various time windows.

2 Principles of registration

Induced polarization I can register either with direct current, or with alternating current. In this case the paper is about interpretation of time spectrum. I have direct current and register in the time domain. I compare two voltages. The first is the voltage of current transmitting and the second is the voltage decay when the current is suspended. I can register spectrally, i.e., registration of voltages in two and more the time windows, or integrally, then I register all extent of voltage decay over all time base. What is characteristic is the voltage decay has an exponential shape.

For quantitative interpretation such characteristics like permeability of sedimentary rocks, prospecting of hydrocarbons in sediments saturated by salt/fresh water and study of influence of various technical and geological factors are the time domain is more useful than the frequency one. I can remember to note authors having study those problems. They are those like DACHNOV, V.N.(1967), DOBRYNIN, V.M. (1988), KOMAROV, S.G., and KOTOV, P.T.(1968), KUZMINA, E. N. and OGIL'VI, A.(1965) and SEIGEL, H. O.(1970) . From Czech authors I have to present very transparent and detailed text-book of KAROUS, M. (1989), where big information of theory, results of modelling and large amount maps of geophysical sections are.

The next important works to variances, frequency and pulse ones, of induced polarization are in KAROUS, M. (1987) and KAROUS, M. (1988).

The elder works are, for example, KNĚZ, J. (1967) and (1975). The mentioned works of Czech authors are very detailed; however, it is mostly about the induced polarization for surface exploration. In spite of all their activities done in the time domain it seems that ways of quantitative interpretation are not exhausted yet. It is about spectral and integral interpretation either in the above domain. The aim of this paper is to present interpretation of the time spectrum and to gain characteristics of the time spectrum like two values of chargeability for invasion zone and non-invaded zone, further, two values of relaxation time for invasion zone and non-invaded

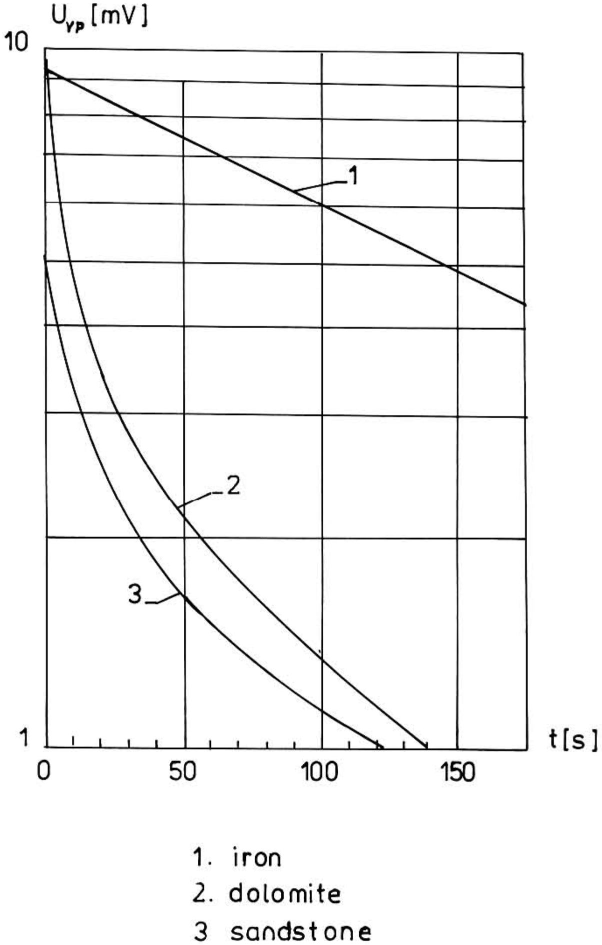

Fig. 1 The relationship of the time spectrum for iron, dolomite and sandstone

zone and, finally, an integral chargeability determined like sum all extent of voltage decay over all time base. The spectral registration in the time domain has really wide possibilities of interpretation for invasion zone and non-invasion zone.

3 Registration of the time spectrum

If you decide to register the time spectrum, you will record time functions like $U_{VP} = f(t)$ or $\kappa_e^* = f(t)$ are. In both cases there – it is an exponential relationship. If it is the one-component bed, you will get a linear relation for the semi-logarithmic system of coordinates. For the two-component or three-component beds the single exponential relationships are added; the final relationship is then non-linear for the semi-logarithmic system of coordinates. Its shape is decreasing. It is distinctly visible in fig.1 made after DACHNOV, V.N. (1967).

This presents different materials: iron and rocks. The curve "iron" is the one-component material. It is gypsum with disseminated iron filings. Gypsum has no single water; the only water is in the crystal lattice and belongs to matrix. Therefore its relation is linear. The sedimentary rocks, like dolomite and sandstone are, have all characteristics being typical for the more-component material. For sure there are two components there. Beside non-invaded zone we can suppose an invasion zone and maybe mud too yet. That is why their relations are not linear.

According to DACHNOV, V.N. (1967) there holds this relation:

$$U_{VP} = \sum_{k=1}^{2} U_k(0) \times \exp\left\{-\left(\frac{t}{\tau_k}\right)\right\}, \qquad (1)$$

where τ_k = the middle time of relaxation of the polarization field for k-th component of rock [s],
$U_k(0)$ = the value of voltage when it holds that $t = 0$ for the k-th component of rock [mV],
U_{VP} = the registered voltage in the selected time window [mV], and
t = the time in the selected window of the time base [s].

Formula (1) is validated by KAROUS, M. (1989), p.176, fig. 100 and formula (182), where is presented how the discharging curve to take on two partial curves apart. The pulse of the direct current will create the voltage acting on the rock. This voltage is defined as follows:

$$U_P = \frac{R \times I}{K_p}, \text{ and} \qquad (2)$$

$$K_p = 4\pi \times L, \qquad (3)$$

where K_p = the constant of the electrode array [m],
L = the tool spacing, i. e., the distance being between A and M electrodes [m],
R = the resistivity of bed [Ωm] and
I = the direct current forming voltage U_P [mA].

If I have log calibrated in the apparent chargeability, I will attain the following formulas:

$$\kappa_e^* = \sum_{i=1}^{2} \kappa_0^{(k)} \times \exp\left\{-\left(\frac{t}{\tau_k}\right)\right\}, \quad (4)$$

$$\kappa_e^* = \frac{U_{VP}}{U_P} \text{ ,and} \quad (5)$$

$$\kappa_0^{(k)} = \frac{U_k(0)}{U_P}, \quad (6)$$

where κ_e^* = the apparent chargeability of bed registered in the selected window, and

$\kappa_0^{(k)}$ = the chargeability of bed for time t = 0 and for the k-th component of bed.

Making spectral registration we are able also to make an integral registration. It means to register all contributions of the only time base. You have simultaneously to integrate all voltage contributions calibrated like the apparent chargeability being between exponential shape of curve and the time base. It presents transformation of irregular plane into the plane of oblong. One of its sides presents the dimension of time base remarked as t_b. The second one – it is the integral value of the apparent chargeability remarked as κ_e(Int). It is well depicted in fig.2.

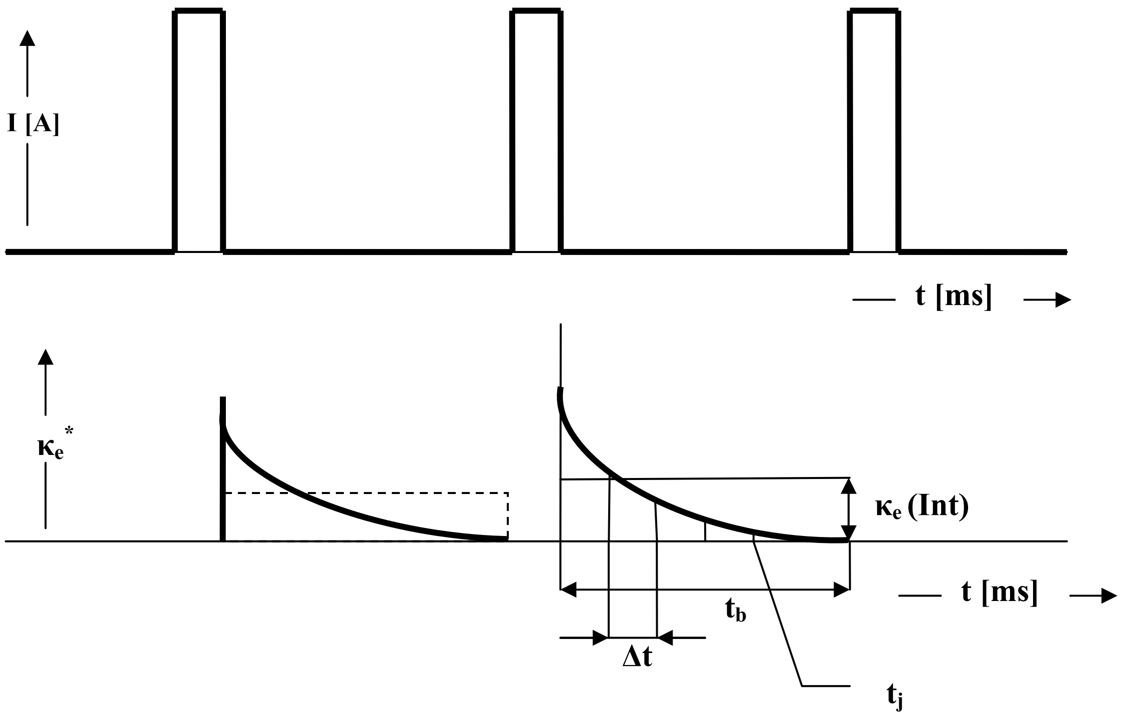

Fig.2 An outline of basic characteristics of the time spectrum

To fig.2 there is little remark. If you calibrate through a ratio of two voltages and it can be, because chargeability is dimensionless factor, you will register directly the apparent chargeability for all extent of time spectrum.

The current pulses on fig.2 are drawn like short; they have but constant width and they are too uniformly high. In well-logging equipments used up to now are pulses being constant and long. Depiction on fig.2 is why in contradiction to formulas (1) and (4), because those suppose variance using long current pulses. Only under the variance it is possible to have all type of induced polarization registered in the domain of discharging curve.

The continuous integration will be replaced by the discrete integration. The formula after the trapezoidal method is following:

$$\kappa_e(\text{Int}) \times t_b = \frac{1}{2} \times \sum_{j=1}^{m-1} \left\{\kappa_e^{(j)} + \kappa_e^{(j+1)}\right\} \times \Delta t, \quad (7)$$

where Δt = the distance between two neighbouring windows of the time base [s] and t_b = the dimension of time base [s].

Now, we can express $\kappa_e(\mathrm{Int})$ from formula (7).

$$\kappa_e(\mathrm{Int}) = \frac{1}{2} \times \left(\frac{\Delta t}{t_b} \right) \times \sum_{j=1}^{m-1} \left\{ \kappa_e^{(j)} + \kappa_e^{(j+1)} \right\}. \quad (8)$$

If we standardized κ_e (Int) with voltage being at end of the current pulse, we should be able to define the factor significant like the charging ability. The one is used in the surface exploration.

Due to formula (8) we can in each point of the borehole depth determine the integral value of the apparent chargeability derived from recorded curves of the time spectrum. That means we have too the continuous curve of the integral values with depth.

4 Analysis and interpretation of the time spectrum

We suppose we have for each i-th depth point m windows when we register dyads (t, κ_e^*). Further, we suppose that each bed consists of two components: invasion zone and non-invaded bed. Mud can be a part of invasion zone when there are caverns. In next formulas there will be used the following indexes:

- The borehole depth: i = 1, 2, 3… n.
- The window of the time spectrum: j = 1, 2, 3…m.
- The component of bed: k = 1, 2.

This chapter is devoted to mathematical decomposition of basic exponential curve on two partial exponential ones that in semi-logarithmical coordinate system are presented like two lines. See, KAROUS, M. (1989), p.176, fig. 100.

For one i-th point of the borehole depth registered in the j-th window of the time spectrum it holds:

$$\kappa_e^{*(i,j)} = \sum_{k=1}^{2} \kappa_{ik}^{(0)} \times \exp\left\{ -\left(\frac{t_j}{\tau_{ik}} \right) \right\} = \kappa_{i1}^{(0)} \times \exp\left\{ -\left(\frac{t_j}{\tau_{i1}} \right) \right\} + \kappa_{i2}^{(0)} \times \exp\left\{ -\left(\frac{t_j}{\tau_{i2}} \right) \right\}, \quad (9)$$

Adjustment of such relation is relatively complicated, but possible. You have dyads $(\kappa_e^{*(i,j)}, t_j)$ and make adjustment with the help of relation (9). The first you have to do is selection of time windows. Basic condition is the windows form arithmetic progression, i.e., it holds that

$$t_{j+1} = t_j + \Delta t. \quad (10)$$

where Δt = the fixed distance between two time windows.

It means you have all observations to make on selected time windows after this condition. Next process is following; we define new variables remarked as X and Y for each i-th depth point, BRONŠTEJN, I.N., and SEMENĎAJEV, K.A. (1963). They are defined like that:

$$Y = \frac{\kappa_e^{*(i,2)}}{\kappa_e^{*(i,j)}}, \; X = \frac{\kappa_e^{*(i,1)}}{\kappa_e^{*(i,j)}}. \tag{11}$$

Relation between them is linear and has following shape:

$$Y = A \times X - B. \tag{12}$$

Thanks to the method of least squares we solve system of equations:

$$\begin{aligned} A \times [X] - B \times m &= [Y] \\ A \times [X^2] - B \times [X] &= [XY]. \end{aligned} \tag{13}$$

Coefficients A and B is defined like this:

$$A = \exp\left\{-\left(\frac{\Delta t}{\tau_{i1}}\right)\right\} + \exp\left\{-\left(\frac{\Delta t}{\tau_{i2}}\right)\right\} = \operatorname{tg}\varphi, \text{ and} \tag{14}$$

$$B = \exp\left\{-\left(\frac{\Delta t}{\tau_{i1}}\right)\right\} \times \exp\left\{-\left(\frac{\Delta t}{\tau_{i2}}\right)\right\}. \tag{15}$$

Symbol φ presents the angle of the line relation intercepting on the vertical axis coefficient B; tg φ is then the gradient of the above line.

It is time to determine characteristics τ_{i1} and τ_{i2}. It holds that

$$\tau_{i1} = \frac{\Delta t}{\ln\left\{\left(\frac{A}{2}\right) + \sqrt{\left(\frac{A}{2}\right)^2 - B}\right\}}, \tag{16}$$

$$\tau_{i2} = \frac{\Delta t}{\ln\left\{\left(\frac{A}{2}\right) - \sqrt{\left(\frac{A}{2}\right)^2 - B}\right\}}. \tag{17}$$

When you express exponential expressions, you will control those relations (14) and (15).

$$e^{-\frac{\Delta t}{\tau_{i1}}} = \left(\frac{A}{2}\right) + \sqrt{\left(\frac{A}{2}\right)^2 - B}, \text{ and } e^{-\frac{\Delta t}{\tau_{i2}}} = \left(\frac{A}{2}\right) - \sqrt{\left(\frac{A}{2}\right)^2 - B}. \tag{18}$$

Now, you have to form new coordinates for evaluation of resting characteristics remarked as $\kappa_{i1}^{(0)}$ and $\kappa_{i2}^{(0)}$, the above coordinates are following, see BRONŠTEJN, I.N., and SEMENĎAJEV, K.A. (1963); it holds again for each i-th depth point:

$$Y^* = \frac{\kappa_e^{*(i,j)}}{e^{-\tau_{i2}\times t_j}} = \kappa_e^{*(i,j)} \times \exp\{\tau_{i2}\times t_j\} \quad \text{and}$$
$$X^* = \frac{e^{-\tau_{i1}\times t_j}}{e^{-\tau_{i2}\times t_j}} = \exp\{(\tau_{i2}-\tau_{i1})\times t_j\}. \tag{19}$$

Relation between them is again linear with following shape:

$$Y^* = \kappa_{i1}^{(0)} \times X^* + \kappa_{i2}^{(0)}. \tag{20}$$

The method of least squares solves then these equations:

$$\kappa_{i1}^{(0)} \times [X] + \kappa_{i2}^{(0)} \times m = [Y]$$
$$\kappa_{i1}^{(0)} \times [X^2] + \kappa_{i2}^{(0)} \times [X] = [XY]. \tag{21}$$

Both coefficients, in this case they are found characteristics, you determine directly. It holds that

$$\kappa_{i1}^{(0)} = \operatorname{tg}\varphi. \tag{22}$$

In this case symbol φ presents the angle of the linear relation intercepting on the vertical axis coefficient $\kappa_{i2}^{(0)}$.

The recorded relation in semi-logarithmic coordinates and having curvilinear shape is divided now into two partial relations; each of them is linear in the system of semi-logarithmic coordinates. Characteristics $\kappa_{i1}^{(0)}$ and $\tau_{i,1}$ belongs to invasion zone that can be influenced too with mud, whereas, characteristics $\kappa_{i2}^{(0)}$ and $\tau_{i,2}$ are for non-invaded zone. The result of interpretation is five continuous curves with depth presenting five characteristics. The fifth is integral characteristic $\kappa_e(\mathrm{Int})$. All these characteristics make possible enlarge information about studied beds.

I should like to recommend more step; to test all depth points according to criterion $\left|\left(\kappa_{i1}^{(0)} - \kappa_{i2}^{(0)}\right)/\kappa_{i1}^{(0)}\right| < 0.05 \cap \left|\left(\tau_{i1} - \tau_{i2}\right)/\tau_{i1}\right| < 0.05$ that determines whether or not the points are one–component material. It can be when either there is no invasion zone, or an extra deep one is there. In such case is k = 1 and you adjust by least squares with the help of the only line for semi-logarithmic coordinates.

There is a next possible characteristic yet; a point of intersection of both partial relations. We start from relation:

$$\kappa_{i1}^{(0)} \times \exp\left\{-\left(\frac{t_i}{\tau_{i1}}\right)\right\} = \kappa_{i2}^{(0)} \times \exp\left\{-\left(\frac{t_i}{\tau_{i2}}\right)\right\}. \tag{23}$$

Thanks to this relation we are able to express time remarked as t_i .

$$-t_i = \left(\frac{\tau_{i1} \times \tau_{i2}}{\tau_{i1} - \tau_{i2}} \right) \times \ln \left(\frac{\kappa_{i1}^{(0)}}{\kappa_{i2}^{(0)}} \right). \tag{24}$$

You will obtain in so way t_i as a continuous curve with depth. The time after (24) reflects position of the boundary being between invasion zone and the non-invaded bed in the time domain. It offers to receive information about depth of invasion zone remarked as D_i . However, it is affair of transformation of the above curve; from the time domain into the distance domain. We have t_i [s] that is transformed with the help of velocity remarked as v [m/s] on the depth of invasion zone D_i [m].

$$t_i / \tau_{ik} = (t_i \times v)/(\tau_{ik} \times v) = x_i / x_{ik}.$$

The key question is what velocity it is and what numeric value has. It ought to be the velocity having relation to relaxation of an induced electric field. It is related to a shift and orientation dipoles. Strictly speaking, it is about velocity of the shift of electric charges in the atoms one another. The mentioned characteristic should be independent of type of environment; simply – constant. However, I am not able to say more correct numeric data of the above velocity in that moment. It would be surely very interesting to interpret factor D_i, possibly, D_i /d, but it asks more investigation. And, moreover, it could be even a false hypothesis.

In spite of that the mentioned question I let opened for next investigators. Interpretation still remains unoccupied.

5 Conclusions

On the base of analysis made before here are these items:

- Through registration of the time spectrum of induced polarization we attain next important characteristics for interpretation. It is, first of all, the middle time of relaxation for invasion zone and non-invaded zone. Further, the starting values of the chargeability when it holds that t = 0 for invasion zone and non-invaded bed. The next characteristic is integral value of the chargeability.
- The bed can be minimally the two-component one. We distinguish usually two components: invasion zone and non-invaded bed.
- The result of interpretation is five continuous curves with the borehole depth. Registration is mostly continuous however, in some case discrete too. For good interpretation of the time spectrum like continuous registration you need to have a wide base of the time spectrum; otherwise it tends to discrete registration. Problem solving is the more complete time analyzer having its time base wide enough like it is for the time spectrum of Impulse Neutron Log used generator of neutrons. On the other side the procedure of interpretation of the time spectrum is useable for Impulse Neutron Log, too.

References

BODMER, R., WARD, S.H., and MORRISON, H.F. *On induced electrical polarization and groundwater*, Geophysics, v. 33, no. 5, 1968, p. 805-821.

BRONŠTEJN, I.N., and SEMENĎAJEV, K.A. *Príručka matematiky pre inžinierov a pre študujúcich na vysokých školách technických*, preklad VIII ruského vydania, SVTL Bratislava, 1963, p.732

CAHYNA, F., MAZAC, O., and VENDHODOVA, D. *Determination of the Extent of Cyanide Contamination by Surface Geoelectrical Methods.* Geotechnical and Environmental Geophysics, Vol 2, Society of Exploration Geophysicists, Tulsa, OK, 1990, pp 97-99.

DACHNOV V.N. *Elektritcheskye i magnitnye metody issledovanya skvazhin-Fundament of theory*, Nedra, Moscow, 1967

DOBRYNIN V.M. *Interpretecya rezultatov geofyzitcheskych issledovanyi neftyanych i gazovych skvazhin*, Handbook, Nedra, Moscow, 1988

KAROUS, M. *Geoelektrické metody průzkumu*, SNTL/ALFA, Praha, 1989, p.423

KAROUS, M. *Vztah frekvenční a impulsní varinary metody vyzvané polarizace*, Geol. průzkum XXIX, No 10(346), 1987, p. 301–304

KAROUS, M. *Polarizovatelnost prostředí jako spojitá funkce časové konstanty*, Seminář Geoelektrické metody používané v ČSSR, 1987, Liblice

KAROUS, M. *Časová závislost polarizovatelnosti a její určování*. Sbor. prací KRB – Rudní prospekce v Jeseníkách. Loučná, 1987

KAROUS, M. *Induced polarization time and frequency chrarakteristics*. Acta Univ. Carol. -Geol. 1, 1988, p. 53–75

KNĚZ, J. *Metodika a technika měření metodou vyzvané polarizace*, Kan. Dis., PřF UK Praha, 1967

KNĚZ, J. *Metoda vyzvané polarizace při vyhledávání a průzkumu zakrytých rudních ložisek*, Habil. práce, PřF UK Praha, 1975

KOMAROV, S.G., and KOTOV, P.T. *On determining formation permeability from induced polarization*: Log Analyst, v. 9, no. 3, 1968, p. 12-17.

KOMAROV, V. A., PISHPAREVA, H., SEMENOV, M. and KHLOPONINA, L. *Theoretical Fundamentals for Interpretation of Survey Data Obtained with the Induced Polarization Method*, Leningrad, Isdatel'stov Nedra, 1966

KUZMINA, E. N. and OGIL'VI, A. *On the Possibility of Using the Induced Polarization Method to Study Ground Water*, Razvedochnaya i Promyslovaya Geofizika, No. 9, 1965, pp 47-69.

LUO, Y., and ZANG G., *Theory and application of spectral induced polarization*: SEG, 171, 1998

SEIGEL, H. O. *Mathematical Formulation and Type Curves for Induced Polarization.* Geophysics, Vol 24, No. 3, 1959, pp 547-565.

SEIGEL, H. O. *The Induced Polarization Method in Mining and Groundwater Geophysics*, Geological Survey Canada, Economic Geology Report 26, 1970

WIGHTMAN, W. E., JALINOOS, F., SIRLES, P., and HANNA, K. *Application of Geophysical Methods to HighwayRelated Problems*, Federal Highway Administration, Central Federal Lands Highway Division, Lakewood, CO, Publication No. FHWA-IF-04-021, September 2003. http://www.cflhd.gov/resources/agm/

WORTHINGTON, P.F. *The relevance of induced polarization to quantitative formation evaluation*, Society of Professional Well Log Analysts, Annual Logging Symposium, 23rd, Corpus Christi, Tex., 1982, Transactions, p. U1-U42, 1982

ZHDANOV M. S. *Generalized effective medium theory of induced polarization*, University of Utah, SEG/New Orleans 2006, Annual Meeting, 2006

Author

[1] RNDr. František Ryšavý, Lesní 3, 695 03 Hodonín, Czech Republic, rysavy.frantisek@seznam.cz