METHOD OF THE CONTROLLED CURRENT REGULATION –PARTIAL CONSTANT k_{EM} FOR ELLIPTICAL ELECTRODES OF THE 2-ELECTRODE MICROLATEROLOG

METODA KONTROLOVANÉ REGULACE PROUDU – DÍLČÍ KONSTANTA k_{EM} PRO ELIPTICKÉ ELEKTRODY 2- ELEKTRODOVÉHO MIKROLATEROLOGU

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Abstract

It is about micro-well-logging of resistivity in the wall of borehole close to the wall. The well-logging uses the focused electric current; the current contours penetrate perpendicularly into the borehole wall. The array of the micro-electrodes can be not only circular, but too elliptical. The current electrode then looks like very thin elliptical/circular contour. The aim of this paper is derivation of formulas needed for calculation of partial constant remarked as k_{EM}. The elliptical electrode array is concentric and is formed by potential electrode shaped as ellipsis and the current electrode as elliptical annulus. The special case is when the current electrode A is simultaneously by the potential electrode M; then it holds that A ≡ M; too holds that E ≡ N where the guard current electrode E serves simultaneously like the potential electrode N. This is characteristic for the 2-electrode Microlaterolog used also for registering of rake angle of beds; the resistivity dipmeter.

Abstrakt

Jedná se o karotážní mikroměření měrného elektrického odporu na stěně vrtu v blízkém okolí stěny vrtu. Měření používá fokusaci elektrického proudu, kdy isolinie proudu vstupují kolmo do stěny vrtu. Mikroelektrodové uspořádání může být nejen kruhové, ale i eliptické. Proudová elektroda potom vypadá jako eliptická/kruhová křivka. Cílem této práce je odvození vzorců potřebných pro výpočet dílčí konstanty označené jako k_{EM}. Eliptické elektrodové uspořádání je koncentrické a je tvořeno potenciálovou elektrodou ve tvaru elipsy a proudovou elektrodou, která má tvar eliptického mezikruží. Zvláštní případ nastane, když proudová elektroda A je zároveň potenciálovou elektrodou M, takže platí A ≡ M. Současně ale platí, že E ≡ N kde stínící proudová elektroda E je zároveň potenciálovou elektrodou N. Toto je charakteristické pro 2elektrodový Mikrolaterolog používaný také pro měření sklonu vrstev; stratametr.

Keywords

current electrode, potential electrode, elliptical annulus, Microlaterolog, well-logging

Klíčová slova

proudová elektroda, potenciální elektroda, eliptické mezikruží, Mikrolaterolog, karotáž

1 Introduction

The potential electrode is an ellipsis surrounded by the current electrode having form of elliptical annulus. The all electrode system is concentric. It is about calculation of partial constant k_{EM}. This is needed for counting the main constant K characterizing the electrode array of Microlaterolog. The analysis of Microlaterolog will be made on basis method of the controlled current regulation, MARUŠIAK, I. et al., (1968) and (1969); it will be in one of the next future papers.

Syntax of the method of the controlled current regulation is based on two fundamental formulas. The first solves calculation of the main constant of the electrode array denoted as K; the second solves calculation of the coefficient of focusing denoted as η. On condition of regulation that holds $U_N = U_M$ those formulas have following form:

$$K = \left\{ \left(k_{AM}^{-1} - k_{BM}^{-1} \right) + k_{EM}^{-1} \times \eta \right\}^{-1} \text{and} \quad \eta = \left(\frac{k_{AN}^{-1} - k_{AM}^{-1}}{k_{EM}^{-1} - k_{EN}^{-1}} \right) + \left(\frac{k_{BM}^{-1} - k_{BN}^{-1}}{k_{EM}^{-1} - k_{EN}^{-1}} \right).$$

The formulas are identical to those for Laterolog, RYŠAVÝ, F (2013). All variables are the partial constants labelled with indexes determining the current and potential electrodes. It is the partial constant k_{EM} used for the 2-electrode Microlaterolog. The central potential electrode has shape as ellipse/circle and is denoted as M; simultaneously it is the current electrode A too. The next current electrode is elliptical/circular annulus denoted as E. Along with that it is the potential electrode N too. This electrode array is characteristic for the 2-electrode Microlaterolog. However, factors K and η can be reduced.

$$K = \left\{ k_{EM}^{-1} \times \eta \right\}^{-1} = k_{AN} \text{ and } \quad \eta = \left(\frac{k_{AN}^{-1}}{k_{EM}^{-1}} \right).$$

It is why that electrode B is located in infinity; so $k_{BM}^{-1} = k_{BN}^{-1} = 0$. Farther, as A≡M and E≡N holds that $k_{AM}^{-1} = k_{EN}^{-1} = 0$. It is evident that important are only two partial constants; k_{EM} and k_{AN}. As derivation of final formula for k_{AN} is topic of other paper, the main aim of this paper is derivation calculation of the partial constant k_{EM}. The detail analyse for Microlaterolog I am going to publish in a paper of next ones. My present work is oriented only and only to calculation of the partial constant k_{EM}.

As you will derive the final formulas for elliptical electrode array you are able to receive the formulas for circular electrode array too. You can only imply condition that both half-axes of ellipsis are equal. So thanks to the elliptical electrode array you can very easy go over to the circular electrode one. This is a mediated way how to get the final formulas for circular electrodes. The next independent way is to derive those formulas in the direct way. In case you have the final formulas for the circular electrodes the same for both ways you receive big probability that you have them rightly derived.

2 Basic metrological characteristics

Fig.1 presents the basic metrological characteristics used in this paper. They are these:

A… the shorter half-axis of ellipsis for the potential electrode [m],
a… the shorter half-axis of the inner ellipsis of the current annulus [m],
B…the longer half-axis of ellipsis for the potential electrode [m],
b… the longer half-axis of the inner ellipsis of the current annulus [m] and
H… the current annulus width [m].

Further, in fig.1 there are remarked the feeding current electrode A, and the guard current electrode E. It presents usual terminology of the current electrodes in focused well-logging methods. Both electrodes are simultaneously the potential ones. It holds that A ≡ M and E ≡ N.

3 Principles of calculation

Unlike the case needed for counting constants k_{AM} and k_{AN} when the current electrode is a central ellipsis, here the current electrode presents elliptical annulus. You integrate in Cartesian variables (x, y). You have to count the potential remarked as U_0 for an arbitrary point of elliptical annulus on all surface of ellipsis presenting the potential electrode. Further, you have to sum simultaneously all potentials U_0 over all points of the surface of annulus; this is new system of Cartesian variables remarked as (k, h) when the centre of system is in the centre of ellipsis. This is well-visible in fig.1. We start of these basic formulas:

$$\rho = \sqrt{x^2 + y^2} \text{ , and} \tag{1}$$

$$S = \pi \times A \times B . \tag{2}$$

Further, we have to define an element of potential U_0:

$$dU_0 = \frac{1}{4\pi} \times \frac{R \times I}{S} \times \frac{dS}{\rho} . \tag{3}$$

Thanks to adjustments from formulas (1) and (2) into formula (3) you receive this expression:

$$dU_0 = \frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{8} \times \left(\frac{2}{\pi}\right) \times \left(\frac{B}{2}\right)^{-1} \times \frac{dx\, dy}{\sqrt{x^2 + y^2}} . \tag{4}$$

Now, we are able to express potential U_0 like double integral in system of Cartesian variables (x, y):

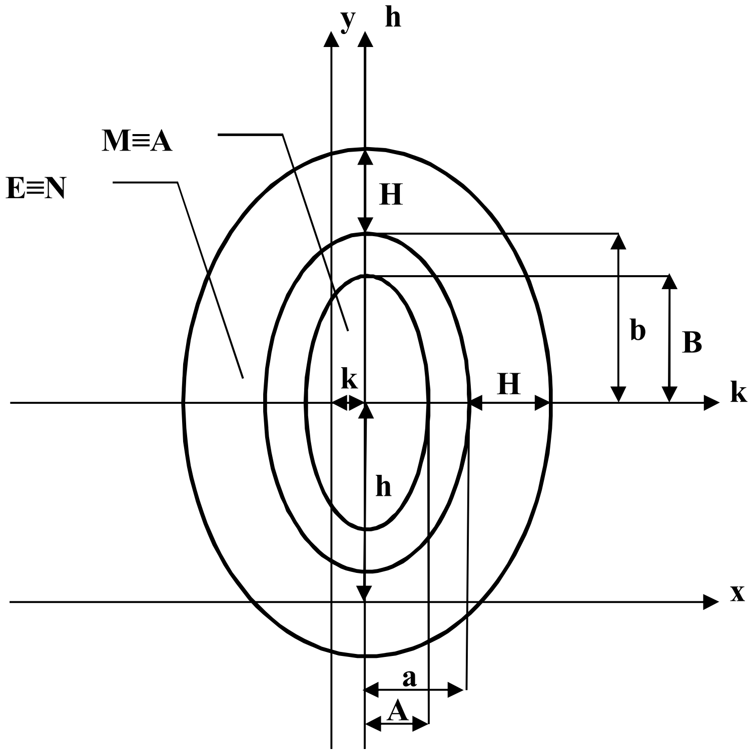

Fig. 1 Depiction of two Cartesian systems of variables, (x, y) and (k, h), needed for describing elliptical array of electrodes

$$U_0 = \frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{8} \times \left(\frac{2}{\pi}\right) \times \left(\frac{B}{2}\right)^{-1} \int\limits_{k-A}^{k+A} \int\limits_{h-B}^{h+B} \frac{dx\, dy}{\sqrt{x^2 + y^2}} . \tag{5}$$

If you make adjustment of this equation, you will attain this expression:

$$U_0 = \frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{8} \times \left(\frac{2}{\pi}\right) \times \left(\frac{B}{2}\right)^{-1} \int\limits_{k-A}^{k+A} \frac{1}{x} \int\limits_{h-B}^{h+B} \frac{dx\, dy}{\sqrt{1 + \left(\frac{y}{x}\right)^2}} . \tag{6}$$

Due to substitution that y = (x×t) you will get that:

$$U_0 = +\frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{8} \times \left(\frac{2}{\pi}\right) \times \left(\frac{B}{2}\right)^{-1} \int\limits_{k-A}^{k+A} \mathrm{Argsinh}\left(\frac{h+B}{x}\right) dx - \frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{8} \times \left(\frac{2}{\pi}\right) \times \left(\frac{B}{2}\right)^{-1} \int\limits_{k-A}^{k+A} \mathrm{Argsinh}\left(\frac{h-B}{x}\right) dx \ . \tag{7}$$

Next substitutions, w = (x/B) and q = (h/B ± 1) ×w^{-1}, provide possibility how to adjust both integrals for the form being convenient for the method per partes. Solution of integrals is this:

$$\begin{aligned} U_0 = &+\frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{4} \times \left(\frac{2}{\pi}\right) \times \left(\frac{A}{B}\right) \times \left(\frac{k}{A} + 1\right) \times \left\{ \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1} \times \left(\frac{\frac{h}{B}+1}{\frac{k}{A}+1}\right)\right] - \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1} \times \left(\frac{\frac{h}{B}-1}{\frac{k}{A}+1}\right)\right]\right\} \\ &-\frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{4} \times \left(\frac{2}{\pi}\right) \times \left(\frac{A}{B}\right) \times \left(\frac{k}{A} - 1\right) \times \left\{ \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1} \times \left(\frac{\frac{h}{B}+1}{\frac{k}{A}-1}\right)\right] - \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1} \times \left(\frac{\frac{h}{B}-1}{\frac{k}{A}-1}\right)\right]\right\} \\ &+\frac{1}{2\pi} \times \frac{(R \times I)}{A} \times \frac{1}{4} \times \left(\frac{2}{\pi}\right) \times \left(\frac{h}{B} + 1\right) \times \left\{ \mathrm{Argsinh}\left[\left(\frac{A}{B}\right) \times \left(\frac{\frac{k}{A}+1}{\frac{h}{B}+1}\right)\right] - \mathrm{Argsinh}\left[\left(\frac{A}{B}\right) \times \left(\frac{\frac{k}{A}-1}{\frac{h}{B}+1}\right)\right]\right\} \end{aligned}$$

$$-\frac{1}{2\pi}\times\frac{(R\times I)}{A}\times\frac{1}{4}\times\left(\frac{2}{\pi}\right)\times\left(\frac{h}{B}-1\right)\times\left\{\ \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)\times\left(\frac{\frac{k}{A}+1}{\frac{h}{B}-1}\right)\right]-\mathrm{Argsinh}\left[\left(\frac{A}{B}\right)\times\left(\frac{\frac{k}{A}-1}{\frac{h}{B}-1}\right)\right]\right\}. \qquad (8)$$

Now, you have new Cartesian variables remarked as (k, h). You are to define an element of voltage dU being defined like this:

$$dU = U_0\times\frac{dS}{S}\text{, and} \qquad (9)$$

$$S=\pi\times[\,(a+H)\times(b+H)-a\times b\,]=\pi\times(a\times b)\times\left[\left(1+\frac{H}{a}\right)\times\left(1+\frac{H}{b}\right)-1\right]. \qquad (10)$$

Element of the annulus surface is expressed as follows:

$$dS=(a\times b)\times\rho\times d\rho\times d\varphi\,. \qquad (11)$$

The unit element of surface is expressed owing to elliptical coordinates:

$$k=a\times\rho\times\cos\varphi\text{, and} \qquad (12)$$

$$h=b\times\rho\times\sin\varphi\,. \qquad (13)$$

The ratio dS/S is determined with the help of formulas (10) and (11):

$$\frac{dS}{S}=\left(\frac{1}{\pi}\right)\times\left[\left(1+\frac{H}{a}\right)\times\left(1+\frac{H}{b}\right)-1\right]^{-1}\times\rho\times d\rho\ d\varphi\,. \qquad (14)$$

Boundaries of new variables (ρ, φ) are these:

$$\varphi\equiv\langle\,0,2\pi\,\rangle\text{, and}$$

$$\rho\equiv\langle\,1,\left[\left(1+\frac{H}{a}\right)^{-2}\times(\cos\varphi)^2+\left(1+\frac{H}{b}\right)^{-2}\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\rangle\,.$$

You have to integrate over all surface of ellipsis. Voltage remarked as U is expressed like this:

$$U=\frac{1}{2\pi}\times\frac{(R\times I)}{A}\times\frac{1}{8}\times\left(\frac{2}{\pi}\right)^2\times\left(\frac{A}{B}\right)\times\left(\left(1+\frac{H}{a}\right)\times\left(1+\frac{H}{b}\right)-1\right)^{-1}\times$$

$$\times\left\{+\int_0^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{a}{A}\times\rho\times\cos\varphi+1\right)\times \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{b}{B}\times\rho\times\sin\varphi+1}{\frac{a}{A}\times\rho\times\cos\varphi+1}\right)\right]d\rho\, d\varphi\right.$$

$$-\int_0^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{a}{A}\times\rho\times\cos\varphi+1\right)\times \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{b}{B}\times\rho\times\sin\varphi-1}{\frac{a}{A}\times\rho\times\cos\varphi+1}\right)\right]d\rho\, d\varphi$$

$$-\int_0^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{a}{A}\times\rho\times\cos\varphi-1\right)\times \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{b}{B}\times\rho\times\sin\varphi+1}{\frac{a}{A}\times\rho\times\cos\varphi-1}\right)\right]d\rho\, d\varphi$$

$$+\int_0^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{a}{A}\times\rho\times\cos\varphi-1\right)\times \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)^{-1}\times\left(\frac{\frac{b}{B}\times\rho\times\sin\varphi-1}{\frac{a}{A}\times\rho\times\cos\varphi-1}\right)\right]d\rho\, d\varphi$$

$$+\int_0^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{b}{B}\times\rho\times\sin\varphi+1\right)\times \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)\times\left(\frac{\frac{a}{A}\times\rho\times\cos\varphi+1}{\frac{b}{B}\times\rho\times\sin\varphi+1}\right)\right]d\rho\, d\varphi$$

$$+\int_0^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{b}{B}\times\rho\times\sin\varphi+1\right)\times \mathrm{Argsinh}\left[\left(\frac{A}{B}\right)\times\left(\frac{\frac{a}{A}\times\rho\times\cos\varphi-1}{\frac{b}{B}\times\rho\times\sin\varphi+1}\right)\right]d\rho\, d\varphi$$

$$-\int_{0}^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{b}{B}\times\rho\times\sin\varphi-1\right)\times\mathrm{Argsinh}\left[\left(\frac{A}{B}\right)\times\left(\frac{\frac{a}{A}\times\rho\times\cos\varphi+1}{\frac{b}{B}\times\rho\times\sin\varphi-1}\right)\right]d\rho\,d\varphi$$

$$+\int_{0}^{2\pi}\int_{\rho_1}^{\rho_2}\rho\times\left(\frac{b}{B}\times\rho\times\sin\varphi-1\right)\times\mathrm{Argsinh}\left[\left(\frac{A}{B}\right)\times\left(\frac{\frac{a}{A}\times\rho\times\cos\varphi-1}{\frac{b}{B}\times\rho\times\sin\varphi-1}\right)\right]d\rho\,d\varphi\ \Bigg\}$$

Next adjustment is about implying new substitutions. With the help of substitutions $w = (\rho - \rho_1)$, $t = (w/w_2)$ and $q = t^{-1}$ for $\rho_1 = 1$, $w_1 = 0$, and $w_2 = \left\{\left[(H/a+1)^{-2}\times(\cos\varphi)^2+(H/b+1)^{-2}\times(\sin\varphi)^2\right]^{-1/2}-1\right\}$ you prepare the integral after ρ for solution with the help of the complex variable. The integral after φ will use substitution $t = \mathrm{tg}\,(\varphi/2)$ and then can be solved again with the help of the complex variable.

The result of double integration is following:

$$U=\frac{1}{2\pi}\times\left(\frac{R\times I}{A}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$

$$\times\left\{+2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\left[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\right]\right. \qquad (15)$$

$$\left.+\left(\frac{a}{b}\right)\times\left[\left(\frac{H}{B}+\frac{b}{B}+1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}+\frac{b}{B}-1\right)\times\mathrm{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}-1\right)^{-1}\right\}\right]\right\}.$$

If you know formula characterizing voltage U, you will be able to express the partial constant k_{EM}.

$$\left(\frac{k}{A}\right)=\frac{2\pi}{F_1+F_2}, \tag{16}$$

$$F_1=+2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$
$$\times\left\{\left[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\right]\right\} \tag{17}$$
and

$$F_2=+\left(\frac{a}{b}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\left(\frac{H}{B}+\frac{b}{B}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}+\frac{b}{B}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}-1\right)^{-1}\right\}\right]\right\}. \tag{18}$$

Both formulas hold under conditions that A < a, and simultaneously B < b.

4 Analysis of the derived formulas

This chapter is about optimum electrode dimensions. It is about whether both electrodes ought to be wide or narrow, possibly whether the one ought to be wide and the second to be narrow and which of them. There are two significant factors; the width of annulus of the current electrode and the width of insulator being between an ellipsis of the potential electrode and the current electrode.

4.1 Wide current annulus

Here holds condition that H >> a > b. Then it holds that:

$$\left(\frac{H}{A}+\frac{a}{A}\right)\approx\left(\frac{H}{A}\right) \text{ and } \left(\frac{H}{B}+\frac{b}{B}\right)\approx\left(\frac{H}{B}\right).$$

In such case you obtain the following expressions for F_1 and F_2:

$$F_1 = +2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right)\times\left(\frac{H}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\left(\frac{H}{A}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}-1\right)^{-1}\right\}\right]\right\},$$

$$F_2 = +\left(\frac{a}{b}\right)\times\frac{\left(\frac{H}{A}\right)^2}{\left(\frac{H}{A}\right)\times\left(\frac{H}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\left(\frac{H}{B}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{H}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{H}{B}-1\right)^{-1}\right\}\right]\right\}.$$

For H→ ∞ it holds that:

$$\left(\frac{H}{A}\pm 1\right)\approx\left(\frac{H}{A}\right) \text{ and } \left(\frac{H}{B}\pm 1\right)\approx\left(\frac{H}{B}\right).$$

You receive that:

$$F_1 = F_2 = 0 \text{ and } F = F_1 + F_2 = 0. \qquad (19)$$

That means the partial constant tends to infinity and voltage on the surface of the potential electrode is zero. This is unpleasant variance excluding optimal dimensions.

4.2 Narrow current annulus

Here holds condition that H<< b < a, what presents that influence of the annulus width is eliminated. In such case there exist conditions:

$$\left(\frac{H}{A}+\frac{a}{A}\right)\approx\left(\frac{a}{A}\right) \text{ and } \left(\frac{H}{B}+\frac{b}{B}\right)\approx\left(\frac{b}{B}\right).$$

It means that holds that (H/A) = (H/B) = 0. It will be used in the next. Now we attain the following expressions:

$$F_1 = +2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$

$$\times\left\{\left[\left(\frac{a}{A}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{a}{A}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}-1\right)^{-1}\right\}\right]\right\}, \text{ and}$$

$$F_2=+\left(\frac{a}{b}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\left(\frac{b}{B}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{b}{B}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{b}{B}-1\right)^{-1}\right\}\right]\right\}.$$

Now, we have to count the limit of expression. Because it is ratio 0/0 we use L´Hospital rule.

$$\lim_{H\to 0}\frac{\left(\frac{H}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}=\lim_{H\to 0}\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}=\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}.$$

So you receive the following expressions for F_1 and F_2.

$$F_1=+2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left\{\left[\left(\frac{a}{A}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{a}{A}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{a}{A}-1\right)^{-1}\right\}\right]\right\}, \text{ and}$$

$$F_2=+\left(\frac{a}{b}\right)\times\frac{\left(\frac{1}{A}\right)\times\left(\frac{a}{A}\right)}{\left(\frac{1}{A}\right)\times\left(\frac{b}{B}\right)+\left(\frac{1}{B}\right)\times\left(\frac{a}{A}\right)}\times\left\{\left[\left(\frac{b}{B}+1\right)\times \mathrm{Argsinh}\left\{\left(\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{b}{B}-1\right)\times \mathrm{Argsinh}\left\{\left(\frac{b}{B}-1\right)^{-1}\right\}\right]\right\}.$$

Narrow insulator (the large potential electrode)

Here hold these conditions:

$\left(\frac{a}{A}\right) \to 1$ and $\left(\frac{b}{B}\right) \to 1$.

We get formulas being given only with both half-axes of the ellipses:

$$F_1 = +4 \times \left(\frac{A}{B}\right)^2 \times \frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right) + \left(\frac{1}{B}\right)} \times \mathrm{Argsinh}\left\{\frac{1}{2} \times \left(\frac{A}{B}\right)^{-1}\right\}, \quad \text{and} \quad F_2 = +2 \times \left(\frac{a}{b}\right) \times \frac{\left(\frac{1}{A}\right)}{\left(\frac{1}{A}\right) + \left(\frac{1}{B}\right)} \times \mathrm{Argsinh}\left(\frac{1}{2}\right). \qquad (20), (21)$$

These are the highest values for both constants F_1 and F_2. The optimal case is when the current annulus is very narrow, however, the potential ellipse is as large as possible, i.e., annulus of insulator between both electrodes is really narrow but functioning too. This is that optimal case.

Wide insulator (the very small potential electrode)

This case is characterized by conditions that:

$\left(\frac{a}{A} \pm 1\right) \approx \left(\frac{a}{A}\right) \to \infty$ and $\left(\frac{b}{B} \pm 1\right) \approx \left(\frac{b}{b}\right) \to \infty$.

These conditions mean that it holds: $F_1 = F_2 = 0$ and $F_1 + F_2 = 0$. This is formula (19). The potential electrode presented is almost point; it has consequence that voltage tends to zero. It is the same effect like it was for the wide current annulus; an unpleasant variance.

The overall analysis allows us to generalize. The surface of the potential electrode ought to be as big as possible; in contrary, the surface of the current electrode should be as small as possible. Then voltage is not zero and you receive its maximal value.

5 Control – deduction of formula for circular electrodes

In such case you transform the elliptical electrode array to the circular electrode one. We implement condition, B = A and b = a, into formulas (17) and (18). Then you get these formulas:

$$F_1 = +2 \times \frac{\left(\frac{H}{A} + \frac{a}{A}\right)}{\left(\frac{H}{A} + 2 \times \frac{a}{A}\right)} \times \left\{\left(\frac{H}{A} + \frac{a}{A} + 1\right) \times \mathrm{Argsinh}\left(\frac{H}{A} + \frac{a}{A} + 1\right)^{-1} - \left(\frac{H}{A} + \frac{a}{A} - 1\right) \times \mathrm{Argsinh}\left(\frac{H}{A} + \frac{a}{A} - 1\right)^{-1}\right\}, \text{ and}$$

$$F_2=+\frac{\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+2\times\frac{a}{A}\right)}\times\left\{\left(\frac{H}{A}+\frac{a}{A}+1\right)\times \text{Argsinh}\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times \text{Argsinh}\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}.$$

For sum of both expressions it is valid that:

$$\left(F_1+F_2\right)=+3\times\frac{\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+2\times\frac{a}{A}\right)}\times\left\{\left(\frac{H}{A}+\frac{a}{A}+1\right)\times \text{Argsinh}\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times \text{Argsinh}\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}. \quad (22)$$

As factors **A**, **a** present now radius of circle we have to implement diameter of circle. We use identities like A = (D/2) and a = (d/2).

$$\frac{k}{\left(\frac{D}{2}\right)}=\left(\frac{k}{D}\right)\times 2=\frac{2\pi}{\left(F_1+F_2\right)}.$$

From this expression you get the following formula:

$$\left(\frac{k}{D}\right)=\frac{2\pi}{F}\text{, and} \quad (23)$$

$$F=2\times\left(F_1+F_2\right)=6\times\frac{\left(\frac{2H}{D}+\frac{d}{D}\right)}{\left(\frac{2H}{D}+2\times\frac{d}{D}\right)}\times\left\{\left(\frac{2H}{D}+\frac{d}{D}+1\right)\times \text{Argsinh}\left(\frac{2H}{D}+\frac{d}{D}+1\right)^{-1}-\left(\frac{2H}{D}+\frac{d}{D}-1\right)\times \text{Argsinh}\left(\frac{2H}{D}+\frac{d}{D}-1\right)^{-1}\right\} \quad (24)$$

where D…the diameter of the circular potential electrode [m] and
d…the inner diameter of the circular current annulus [m].

The formula holds for D < d. This is formula derived for the circular annulus. It is partial case of the elliptical one. As I made, for my own control, the direct way of derivation of final formula for circular electrodes too, I can say the formula is derived rightly. Consequence of that is both formulas for circular and elliptical electrodes are rightly derived.

6 Derivation of formula for the common potential and current electrodes; E ≡ N

It is case of the 2-electrode Microlaterolog. The 2-electrode elliptical Proximity log it can be too. It holds in both cases that E≡N and A≡M. Let's return again to basic formulas, formulas (17) and (18).

$$F_1 = +2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times$$

$$\times\left\{\left[\left(\frac{H}{A}+\frac{a}{A}+1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}+1\right)^{-1}\right\}-\left(\frac{H}{A}+\frac{a}{A}-1\right)\times\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}-1\right)^{-1}\right\}\right]\right\}, \text{ and}$$

$$F_2 = +\left(\frac{a}{b}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\left(\frac{H}{B}+\frac{b}{B}+1\right)\times\text{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}+1\right)^{-1}\right\}-\left(\frac{H}{B}+\frac{b}{B}-1\right)\times\text{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}-1\right)^{-1}\right\}\right]\right\}.$$

If it holds that the only electrode shaped as elliptical annulus exists, then such electrode presents both potential and current one. An electrode spacing between electrodes E and N is zero, i.e. $\overline{\text{EN}} = 0$. As A = 0 and B = 0 too, it is possible to adjust both functions when you use the following conditions:

$$\left(\frac{H}{A}+\frac{a}{A}\pm 1\right)\to\left(\frac{H}{A}+\frac{a}{A}\right),\ \left(\frac{H}{B}+\frac{b}{B}\pm 1\right)\to\left(\frac{H}{B}+\frac{b}{B}\right).$$

Both functions will be changed as follows:

$$F_1 = +2\times\left(\frac{A}{B}\right)^2\times\left(\frac{a}{b}\right)^{-1}\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)^2}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}\right)^{-1}\right\}-\text{Argsinh}\left\{\left(\frac{A}{B}\right)^{-1}\times\left(\frac{H}{A}+\frac{a}{A}\right)^{-1}\right\}\right]\right\} = 0, \text{ and}$$

$$F_2 = +\left(\frac{a}{b}\right)\times\frac{\left(\frac{H}{A}\right)\times\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)}{\left(\frac{H}{A}+\frac{a}{A}\right)\times\left(\frac{H}{B}+\frac{b}{B}\right)-\left(\frac{a}{A}\right)\times\left(\frac{b}{B}\right)}\times\left\{\left[\text{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}\right)^{-1}\right\}-\text{Argsinh}\left\{\left(\frac{H}{B}+\frac{b}{B}\right)^{-1}\right\}\right]\right\} = 0.$$

Let's return to formula (16).

$$\left(\frac{k}{A}\right)=\frac{2\pi}{F_1+F_2},$$

If $\overline{EN}=0$ then holds that A = 0 and B = 0 what presents that $F_1 = 0$, $F_2 = 0$, too. Their sum is zero; $(F_1 + F_2) = 0$. In such case the constant k_{EN} goes to infinity, i.e. $k_{EN} \rightarrow \infty$. Its reciprocal value is equal to zero; $k_{EN}^{-1}=0$ and it is why that U = 0. Just this confirms condition needed for reduced factors K and η given in Introduction.

7 Conclusions

Analysis of derived formulas yields the following conclusions:

- The partial constant k_{EM} of the elliptical electrodes can be exactly counted, because all needed characteristics of the electrode array are well-measurable.
- The elliptical array of electrodes in comparison to circular one has two components remarked as F_1 and F_2 either for one of half-axes.
- It is possible to get the final formula for the circular electrodes too, thanks to implication of condition equality for both half-axes ellipses.
- If it is that holds that E ≡ N, the voltage of the annulus electrode is zero and the partial constant tends to infinity. Consequence of that is the reciprocal value of the partial constant is equal to zero; $k_{EN}^{-1}=0$; the effect is independent on the electrode shape.
- It holds that the surface of the potential electrode being in the centre of the electrode array should be as big as possible, whereas, the surface of annulus of the current one must be as narrow as possible. It is an optimal electrode array. The current electrode looks then like an elliptic/circular contour.

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