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METHOD OF THE CONTROLLED CURRENT REGULATION – CALCULATION OF PARTIAL CONSTANTS k_{AM} AND k_{AN} FOR ELLIPTICAL ELECTRODES OF MICROLATEROLOG

METODA KONTROLOVANÉ REGULACE PROUDU – VÝPOČET DÍLČÍCH KONSTANT k_{AM}, k_{AN} PRO ELIPTICKÉ ELEKTRODY MIKROLATEROLOGU

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Abstract

It is about micro-well-logging of resistivity in the wall of borehole close to the wall. The well-logging uses the focused electric current; the current contours penetrate perpendicularly into the borehole wall. The array of the micro-electrodes can be not only circular, but too elliptical. The current electrode then looks like very thin elliptical/circular contour. The aim of this paper is derivation of formulas needed for calculation of partial constant denoted as k_{AM} and k_{AN}. This paper explains action of the current electrode formed like ellipsis on the potential one having its form like an elliptical annulus surrounding the current one. The constant has two components; it presents the big and small half-axes. The circular electrode array is partial case of the elliptical one. If the only common electrode exists being simultaneously the potential and current ones, the voltage will be zero.

Abstrakt

Jedná se o karotážní mikroměření měrného elektrického odporu na stěně vrtu v blízkém okolí stěny vrtu. Měření používá fokusaci elektrického proudu, kdy isolinie proudu vstupují kolmo do stěny vrtu. Mikroelektrodové uspořádání může být nejen kruhové, ale i eliptické. Proudová elektroda potom vypadá jako eliptická/kruhová křivka. Cílem této práce je odvození vzorců potřebných pro výpočet dílčí konstanty označené jako k_{AM} a k_{AN}. Tato práce vysvětluje působení proudové elektrody ve tvaru elipsy na potenciálovou elektrodu, která je tvarovaná jako eliptické mezikruží obklopující proudovou elektrodu. Konstanta má dvě složky; jedná se o hlavní a vedlejší poloosu. Kruhové uspořádání elektrod je pak dílčím případem eliptického uspořádání. Existuje-li jedna společná elektroda, která je zároveň potenciální i proudová, má nulové napětí.

Keywords

elliptical array, the current electrode, the potential electrode, ellipsis, annulus, Microlaterolog, well-logging

Klíčová slova

eliptické uspořádání, proudová elektroda, potenciální elektroda, elipsa, mezikruží, Mikrolaterolog, karotáž

1 Introduction

If one is going to explain principles of Microlaterolog on basis the method of the controlled current regulation, MARUŠIAK, I. et al., (1968) and (1969), it has to anticipate derivation of the partial constants like k_{AM}, and k_{AN} are. As electrodes have not cylindrical surface it is not possible to use the derived formulas of paper RYŠAVÝ, F. (2013), but they must be all newly derived. The electrode array is presented the current electrode is formed like ellipsis and the potential electrode having form of elliptical annulus. The above system is concentric.

As each of two mentioned electrodes can change its form from ellipsis to circle there exist four possible combinations: ellipsis and elliptical annulus, ellipsis and circular annulus, circle and elliptical annulus and, finally, circle and circular annulus. The last one is most known. All formulas valid for each of combinations can be derived from the basic formula for the first combination. What is important is the potential electrode is always elliptical/circular annulus and the current electrode presents ellipsis/circle.

Syntax of the method of the controlled current regulation is based on two fundamental formulas. The first solves calculation of the main constant of the electrode array denoted as K; the second solves calculation of the coefficient of focusing denoted as η. On condition of regulation that holds $U_N = U_M$ those formulas have following form:

$$K = \left\{ k_{AM}^{-1} + k_{EM}^{-1} \times \eta \right\}^{-1} \text{and} \quad \eta = \left(\frac{k_{AN}^{-1} - k_{AM}^{-1}}{k_{EM}^{-1} - k_{EN}^{-1}} \right) .$$

All variables are the partial constants labelled with indexes determining the current and potential electrodes. In this paper there are the partial constants denoted as k_{AM} and k_{AN}. The next constants k_{EM} and k_{EN} are solved in other paper. The central current electrode has shape as ellipse/circle and is denoted as A, whereas the potential one is elliptical/circular annulus denoted as M or N. The formulas are identical to those for Laterolog, RYŠAVÝ, F (2013). The detail analyse for Microlaterolog I am going to publish in a next paper. My present work is oriented only and only to calculation of the partial constants k_{AM} and k_{AN}.

As you will derive the final formulas for elliptical electrode array you are able to receive the formulas for circular electrode array too. You can only imply condition that both half-axes of ellipsis are equal. So thanks to the elliptical electrode array you can very easy go over to the circular electrode one. This is a mediated way how to get the final formulas for circular electrodes. The next independent way is to derive those formulas for circular ones in the direct way. In case you have the final formulas for the circular electrodes the same for both ways you receive big probability that you have them rightly derived.

2 Basic metrological characteristics

Fig.1 presents the basic metrological characteristics used in this paper. They are these:

A… the shorter half-axis of ellipsis for the current electrode [m],

a… the shorter half-axis of the inner ellipsis of the potential annulus [m],

B…the longer half-axis of ellipsis for the current electrode [m],

b… the longer half-axis of the inner ellipsis of the potential annulus [m] and
H… the potential annulus width [m].
Further, in fig.1 there is denoted the feeding current electrode as A; the potential electrodes are M/N. It presents usual terminology of the current electrodes in focused well-logging methods. Special case is when holds that A ≡ M.

3 Derivation of the basic formula

After fig.1 it is evident you have to start from the potential of elliptical annulus evoked by an arbitrary point on the surface of the current electrode. This potential will be denoted as U_0.

$$dU_0 = dU_2 - dU_1 = \frac{1}{4\pi} \times (R \times I) \times \frac{1}{\sqrt{x^2 + y^2}} \times \left[\frac{dS}{S_2} - \frac{dS}{S_1} \right], \quad (1)$$

$$S_2 = \pi \times (a \times b), \text{ and} \quad (2)$$

$$S_1 = \pi \times (a + H) \times (b + H) = \pi \times (a \times b) \times \left(\frac{H}{a} + 1 \right) \times \left(\frac{H}{b} + 1 \right). \quad (3)$$

Equation of the inner ellipsis of annulus is this:

$$\frac{(x-k)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1. \quad (4)$$

Equation of the outer ellipsis of annulus is following:

$$\frac{(x-k)^2}{(a+H)^2} + \frac{(y-k)^2}{(b+H)^2} = 1. \quad (5)$$

Now, you are to implement the elliptical coordinates:

$$x = a \times \rho \times \cos\varphi, \text{ and} \quad (6)$$

$$y = b \times \rho \times \sin\varphi. \quad (7)$$

We are able to express an element of the surface denoted as dS:

$$dS = (a \times b) \times \rho \times d\rho\, d\varphi. \quad (8)$$

Further, we have to write down ratios (dS/S_2) and (dS/S_1):

$$\frac{dS}{S_2} = \frac{1}{2} \times \left(\frac{2}{\pi} \right) \times \rho \times d\rho\, d\varphi, \text{ and} \quad (9)$$

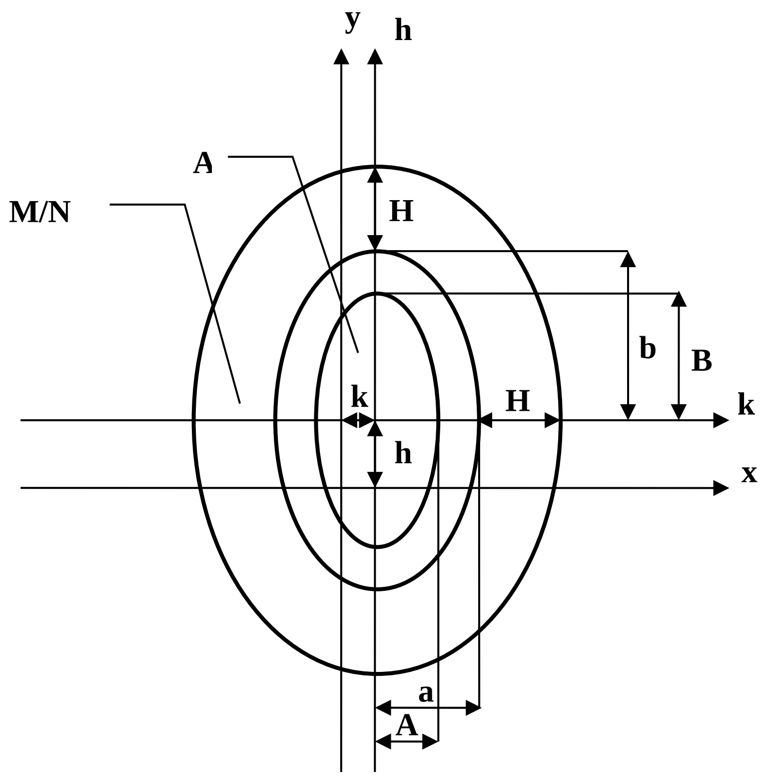

Fig.1 Depiction of two Cartesian systems of variables, (x, y) and (k, h), needed for describing elliptical array of electrodes

$$\frac{dS}{S_1}=\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{H}{a}+1\right)^{-1}\times\left(\frac{H}{b}+1\right)^{-1}\times\rho\times d\rho\, d\varphi\,. \tag{10}$$

You have to express the following expression, as well:

$$\frac{1}{\sqrt{x^2+y^2}}=\frac{1}{\rho}\times\frac{1}{\sqrt{a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2}}\,. \tag{11}$$

Now, we are able to get expression for dU_0:

$$dU_0=\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{2}\right)\times\frac{1}{\sqrt{a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2}}\times\left[1-\left(\frac{H}{a}+1\right)^{-1}\times\left(\frac{H}{b}+1\right)^{-1}\right]d\rho\, d\varphi\,. \tag{12}$$

You have to determine the boundaries of integration after φ and ρ. For the variable φ they are these:

$\varphi\equiv\langle 0,2\pi\rangle$.

The boundaries of ρ for the outer ellipsis of annulus are these:

$$\rho_{22}=\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]^{-1}\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{k}{a}\right)\times\cos\varphi+\left(\frac{H}{b}+1\right)^{-2}\times\left(\frac{h}{b}\right)\times\sin\varphi\right]+\right.$$
$$\left.+\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^2\right\}^{\frac{1}{2}}\right\} \text{ and} \tag{13}$$

$$\rho_{21}=\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]^{-1}\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{k}{a}\right)\times\cos\varphi+\left(\frac{H}{b}+1\right)^{-2}\times\left(\frac{h}{b}\right)\times\sin\varphi\right]-\right.$$
$$\left.-\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^2\right\}^{\frac{1}{2}}\right\}. \tag{14}$$

The boundaries of ρ for the inner ellipsis of annulus are following:

$$\rho_{12}=\left[\left(\frac{k}{a}\right)\times\cos\varphi+\left(\frac{h}{b}\right)\times\sin\varphi\right]+\sqrt{1-\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^{2}}\text{, and} \tag{15}$$

$$\rho_{11}=\left[\left(\frac{k}{a}\right)\times\cos\varphi+\left(\frac{h}{b}\right)\times\sin\varphi\right]-\sqrt{1-\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^{2}}. \tag{16}$$

The voltage U_0 is expressed as follows:

$$\begin{aligned}U_0=&\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{2}\right)\int_0^{2\pi}\int_{\rho_{11}}^{\rho_{12}}\frac{d\rho\, d\varphi}{\sqrt{a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2}}\\&-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{2}\right)\times\left(\frac{H}{a}+1\right)^{-1}\times\left(\frac{H}{b}+1\right)^{-1}\int_0^{2\pi}\int_{\rho_{21}}^{\rho_{22}}\frac{d\rho\, d\varphi}{\sqrt{a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2}}.\end{aligned} \tag{17}$$

We integrate after ρ. In such case you will get the following expression:

$$\begin{aligned}U_0=&\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{2}\right)\int_0^{2\pi}\left[a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\times\left\{1-\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^{2}\right\}^{\frac{1}{2}}-\\&-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{2}\right)\times\left(\frac{H}{a}+1\right)^{-1}\times\left(\frac{H}{b}+1\right)^{-1}\int_0^{2\pi}\left[a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\times\\&\times\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]^{-1}\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]-\right.\\&\left.-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^{2}\right\}^{\frac{1}{2}}d\varphi\ .\end{aligned} \tag{18}$$

Integrals in question are two; the less and the more complicated. At first we are to solve the more complicated one needed for counting of both integrals, because the less complicated one is only partial case of the more complicated:

$$\int_0^{2\pi} \left[a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\times\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]^{-1}\times$$

$$\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^2\right\}^{\frac{1}{2}}d\varphi=$$

$$=2\int_0^{\frac{\pi}{2}} \left[a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\times\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]^{-1}\times$$

$$\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^2\right\}^{\frac{1}{2}}d\varphi+$$

$$+2\int_0^{\frac{\pi}{2}} \left[a^2\times(\sin\varphi)^2+b^2\times(\cos\varphi)^2\right]^{-\frac{1}{2}}\times\left[\left(\frac{H}{a}+1\right)^{-2}\times(\sin\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\cos\varphi)^2\right]^{-1}\times$$

$$\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\sin\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\cos\varphi)^2\right]-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\sin\varphi-\left(\frac{k}{a}\right)\times\cos\varphi\right]^2\right\}^{\frac{1}{2}}d\varphi. \quad (19)$$

We implement substitution tg (φ/2) = t and both integrals solve with the help of the complex variable. The result is that:

$$\int_0^{2\pi} \left[a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\times\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]^{-1}\times$$

$$\times\left\{\left[\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2+\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2\right]-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{H}{b}+1\right)^{-2}\times\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^2\right\}^{\frac{1}{2}}d\varphi=$$

$$= 6\times\left(\frac{\pi}{2}\right)\times\left[\ \frac{1}{b}\times\left(\frac{H}{b}+1\right)\times\sqrt{1-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{k}{a}\right)^{2}}+\frac{1}{a}\times\left(\frac{H}{a}+1\right)\times\sqrt{1-\left(\frac{H}{b}+1\right)^{-2}\times\left(\frac{h}{b}\right)^{2}}\ \right]. \quad (20)$$

If you install condition into formula (20) that H = 0, you will be able to express the less complicated integral needed for counting U_0:

$$\int_{0}^{2\pi}\left[a^2\times(\cos\varphi)^2+b^2\times(\sin\varphi)^2\right]^{-\frac{1}{2}}\times\left\{1-\left[\left(\frac{h}{b}\right)\times\cos\varphi-\left(\frac{k}{a}\right)\times\sin\varphi\right]^2\right\}^{\frac{1}{2}}d\varphi = 6\times\left(\frac{\pi}{2}\right)\times\left[\frac{1}{b}\times\sqrt{1-\left(\frac{k}{a}\right)^2}+\frac{1}{a}\times\sqrt{1-\left(\frac{h}{b}\right)^2}\ \right]. \quad (21)$$

Now, you can determine U_0:

$$U_0=\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times 3\times\left[\left(\frac{a}{b}\right)\times\sqrt{1-\left(\frac{k}{a}\right)^2}+\sqrt{1-\left(\frac{h}{b}\right)^2}\ \right]$$
$$-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times 3\times\left[\left(\frac{a}{b}\right)\times\left(\frac{H}{a}+1\right)^{-1}\times\sqrt{1-\left(\frac{H}{a}+1\right)^{-2}\times\left(\frac{k}{a}\right)^2}+\left(\frac{H}{b}+1\right)^{-1}\times\sqrt{1-\left(\frac{H}{b}+1\right)^{-2}\times\left(\frac{h}{b}\right)^2}\ \right]. \quad (22)$$

We have to determine an element of voltage denoted as dU for the system of new Cartesian coordinates (k, h).

$$dU = U_0\times\frac{dS}{S}. \quad (23)$$

For the surface of ellipsis it holds that:

$$S=\pi\times(A\times B). \quad (24)$$

Applying new elliptical coordinates you will obtain expression for dS:

$$dS = (A\times B)\times\rho\times d\rho\, d\varphi. \quad (25)$$

$$k = A\times\rho\times\cos\varphi\text{, and} \quad (26)$$

$$h = B\times\rho\times\sin\varphi. \quad (27)$$

Now, you can express ratio dS/S:

$$\frac{dS}{S}=\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\rho\times d\rho\, d\varphi. \quad (28)$$

The boundaries of integration for ρ are these: ρ ≡ < 0, 1>; for φ there hold the boundaries: φ ≡ < 0, 2π >. We can define expression for voltage U:

$$U=+\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times 3\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\int_0^{2\pi}\int_0^1 \rho\times\sqrt{1-\left(\frac{A}{a}\right)^2\times\rho^2\times(\cos\varphi)^2}\;d\rho\,d\varphi$$

$$+\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times 3\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\int_0^{2\pi}\int_0^1 \rho\times\sqrt{1-\left(\frac{B}{b}\right)^2\times\rho^2\times(\sin\varphi)^2}\;d\rho\,d\varphi \tag{29}$$

$$-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times 3\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left(\frac{H}{a}+1\right)^{-1}\int_0^{2\pi}\int_0^1 \rho\times\sqrt{1-\left(\frac{A}{a}\right)^2\times\left(\frac{H}{a}+1\right)^{-2}\times\rho^2\times(\cos\varphi)^2}\;d\rho\,d\varphi$$

$$-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times 3\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{H}{b}+1\right)^{-1}\int_0^{2\pi}\int_0^1 \rho\times\sqrt{1-\left(\frac{B}{b}\right)^2\times\left(\frac{H}{b}+1\right)^{-2}\times\rho^2\times(\sin\varphi)^2}\;d\rho\,d\varphi\ .$$

Integration after ρ thanks to substitution $t = (H/a + 1)^{-1} \times (A/a) \times \cos\varphi \times \rho$ is affair of elementary functions. You will receive this expression:

$$U=+\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\int_0^{2\pi}\sqrt{1-\left(\frac{A}{a}\right)^2\times(\cos\varphi)^2}\,d\varphi+\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\int_0^{2\pi}\sqrt{1-\left(\frac{B}{b}\right)^2\times(\sin\varphi)^2}\,d\varphi$$

$$-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left(\frac{H}{a}+1\right)^{-1}\int_0^{2\pi}\sqrt{1-\left(\frac{A}{a}\right)^2\times\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2}\;d\varphi \tag{30}$$

$$-\frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\frac{1}{2}\times\left(\frac{2}{\pi}\right)\times\left(\frac{H}{b}+1\right)^{-1}\int_0^{2\pi}\sqrt{1-\left(\frac{B}{b}\right)^2\times\left(\frac{H}{b}+1\right)^{-2}\times(\sin\varphi)^2}\,d\varphi\ .$$

We solve this integral:

$$\int_0^{2\pi}\sqrt{1-\left(\frac{A}{a}\right)^2\times\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2}\,d\varphi\ =4\int_0^{\frac{\pi}{2}}\sqrt{1-\left(\frac{A}{a}\right)^2\times\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2}\,d\varphi\,.$$

Due to substitution cos φ = sin α you will get this solution:

$$\int_0^{2\pi}\sqrt{1-\left(\frac{A}{a}\right)^2\times\left(\frac{H}{a}+1\right)^{-2}\times(\cos\varphi)^2}\,d\varphi\ =4\times\mathrm{E}\left\{\left(\frac{A}{a}\right)\times\left(\frac{H}{a}+1\right)^{-1}\right\}. \tag{31}$$

With the help of condition that H = 0 you receive the next integral:

$$\int_0^{2\pi} \sqrt{1-\left(\frac{A}{a}\right)^2 \times (\cos\varphi)^2}\, d\varphi = 4\times \mathrm{E}\left\{\left(\frac{A}{a}\right)\right\}. \tag{32}$$

The next integral is again elliptical one:

$$\int_0^{2\pi} \sqrt{1-\left(\frac{B}{b}\right)^2 \times \left(\frac{H}{b}+1\right)^{-2} \times (\sin\varphi)^2}\, d\varphi = 4\int_0^{\frac{\pi}{2}} \sqrt{1-\left(\frac{B}{b}\right)^2 \times \left(\frac{H}{b}+1\right)^{-2} \times (\sin\varphi)^2}\, d\varphi == 4\times \mathrm{E}\left\{\left(\frac{B}{b}\right)\times\left(\frac{H}{b}+1\right)^{-1}\right\}. \tag{33}$$

And again with the help of condition that H = 0 you attain that:

$$\int_0^{2\pi} \sqrt{1-\left(\frac{B}{b}\right)^2 \times (\sin\varphi)^2}\, d\varphi = 4\times \mathrm{E}\left\{\left(\frac{B}{b}\right)\right\}. \tag{34}$$

Finally, we obtain the formula for voltage U:

$$U = \frac{1}{2\pi}\times\left(\frac{R\times I}{a}\right)\times\left\{2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[\mathrm{E}\left\{\frac{A}{a}\right\}-\left(\frac{H}{a}+1\right)^{-1}\times\mathrm{E}\left\{\left(\frac{A}{a}\right)\times\left(\frac{H}{a}+1\right)^{-1}\right\}\right]+2\times\left(\frac{2}{\pi}\right)\times\left[\mathrm{E}\left\{\frac{B}{b}\right\}-\left(\frac{H}{b}+1\right)^{-1}\times\mathrm{E}\left\{\left(\frac{B}{b}\right)\times\left(\frac{H}{b}+1\right)^{-1}\right\}\right]\right\}. \tag{35}$$

We can now define the formula for the partial constants k_{AM} and k_{AN}:

$$\left(\frac{k}{a}\right) = \frac{2\pi}{F_1+F_2}, \tag{36}$$

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[\mathrm{E}\left\{\frac{A}{a}\right\}-\left(\frac{H}{a}+1\right)^{-1}\times\mathrm{E}\left\{\left(\frac{A}{a}\right)\times\left(\frac{H}{a}+1\right)^{-1}\right\}\right], \text{ for } A < a \text{ and} \tag{37}$$

$$F_2 = +2\times\left(\frac{2}{\pi}\right)\times\left[\mathrm{E}\left\{\frac{B}{b}\right\}-\left(\frac{H}{b}+1\right)^{-1}\times\mathrm{E}\left\{\left(\frac{B}{b}\right)\times\left(\frac{H}{b}+1\right)^{-1}\right\}\right] \text{ for } B < b\,. \tag{38}$$

4 Analysis of the derived formulas

This chapter is about optimal dimensions of both electrodes. From formulas (37) and (38) there result out these inequalities:

$$\left(\frac{A}{a}\right)\langle 1 \langle \left(\frac{H}{a}+1\right);\ \left(\frac{H}{a}+1\right)^{-1}\langle 1, \quad \text{and}$$

$$\left(\frac{B}{b}\right)\langle 1 \langle \left(\frac{H}{b}+1\right);\ \left(\frac{H}{b}+1\right)^{-1}\langle 1.$$

We distinguish wide and narrow elliptical annulus. The following analysis directs to optimal dimensions of the current and potential electrodes.

4.1 Wide elliptical potential annulus

This is the case when it holds that H >> a > b. We are allowed to apply the following conditions:

$$\left(\frac{H}{a}+1\right)\to\left(\frac{H}{a}\right) \text{and} \left(\frac{H}{b}+1\right)\to\left(\frac{H}{b}\right).$$

After implement of those conditions you attain formulas F_1 and F_2 as follows:

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[\ \mathrm{E}\left\{\frac{A}{a}\right\}-\left(\frac{H}{a}\right)^{-1}\times\mathrm{E}\left\{\left(\frac{A}{a}\right)\times\left(\frac{H}{a}\right)^{-1}\right\}\ \right], \text{and} \tag{39}$$

$$F_2 = +2\times\left(\frac{2}{\pi}\right)\times\left[\ \mathrm{E}\left\{\frac{B}{b}\right\}-\left(\frac{H}{b}\right)^{-1}\times\mathrm{E}\left\{\left(\frac{B}{b}\right)\times\left(\frac{H}{b}\right)^{-1}\right\}\ \right]. \tag{40}$$

If it holds that (H/a) → ∞ and (H/b) → ∞, too, it will present that $(H/a)^{-1}$ → 0 and $(H/b)^{-1}$ → 0, as well. This carries consequence that:

$$\mathrm{E}\left\{\left(\frac{A}{a}\right)\times\left(\frac{H}{a}\right)^{-1}\right\}=\mathrm{E}\left\{\left(\frac{B}{b}\right)\times\left(\frac{H}{b}\right)^{-1}\right\}\to\mathrm{E}\{0\}=\frac{\pi}{2}.$$

The formulas (39) and (40) will get simpler form:

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[\ \mathrm{E}\left\{\frac{A}{a}\right\}-0\times\frac{\pi}{2}\ \right]=+2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\mathrm{E}\left\{\frac{A}{a}\right\}, \text{and} \tag{41}$$

$$F_2 = +2\times\left(\frac{2}{\pi}\right)\times\left[\ \mathrm{E}\left\{\frac{B}{b}\right\}-0\times\frac{\pi}{2}\ \right]=+2\times\left(\frac{2}{\pi}\right)\times\mathrm{E}\left\{\frac{B}{b}\right\}. \tag{42}$$

These are real formulas. Now it depends only on ratios A/a and B/b. We have to distinguish between wide and narrow insulator between both electrodes.

Narrow insulator (the large current electrode)

In such case it is valid that A → a, and simultaneously B → b. This can be expressed like that:

$$\left(\frac{A}{a}\right)=\left(\frac{B}{b}\right)\rightarrow 1.$$

That means we will have that:

$$\mathrm{E}\left\{\frac{A}{a}\right\}=\mathrm{E}\left\{\frac{B}{b}\right\}\rightarrow \mathrm{E}\{1\}=1.$$

Then it is possible to write:

$$F_1=+2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right),\text{ and} \qquad (43)$$

$$F_2=+2\times\left(\frac{2}{\pi}\right). \qquad (44)$$

Wide insulator (the very small current electrode – almost the point one)

Then it holds that A << a, and B << b. These conditions can be written down like:

$$\left(\frac{A}{a}\right)=\left(\frac{B}{b}\right)\rightarrow 0.$$

For elliptical integrals it is possible to apply easing conditions:

$$\mathrm{E}\left\{\frac{A}{a}\right\}=\mathrm{E}\left\{\frac{B}{b}\right\}\rightarrow \mathrm{E}\{0\}=\frac{\pi}{2}.$$

In such case it holds:

$$F_1=+2\times\left(\frac{2}{\pi}\right)\times\left(\frac{\pi}{2}\right)\times\left(\frac{a}{b}\right)=2\times\left(\frac{a}{b}\right),\text{ and} \qquad (45)$$

$$F_2=+2\times\left(\frac{2}{\pi}\right)\times\left(\frac{\pi}{2}\right)=2. \qquad (46)$$

Both variances are the optimal; however, lower values are for the narrow insulator, which means the large current electrode. For construction of an optimal electrode system you must account with both variances simultaneously!

4.2 Narrow elliptical potential annulus

This is the case when it holds that H << b < a. We are allowed to write down that like:

$\left(\frac{H}{a}+1\right)\rightarrow 1$ and $\left(\frac{H}{b}+1\right)\rightarrow 1$.

The result is that:

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[\ \mathrm{E}\left\{\frac{A}{a}\right\}-\mathrm{E}\left\{\frac{A}{a}\right\}\ \right]=0\text{, and} \tag{47}$$

$$F_2 = +2\times\left(\frac{2}{\pi}\right)\times\left[\ \mathrm{E}\left\{\frac{B}{b}\right\}-\mathrm{E}\left\{\frac{B}{b}\right\}\ \right]=0\,. \tag{48}$$

The consequence of that is that $(F_1 + F_2) = 0$. That means (k/a) → ∞. It holds that k → ∞ and k^{-1}→ 0. The result is U → 0. This variant is not at all convenient for construction of the potential electrode. Here bigness of current electrode does not play a role.

5 Analysis after the shape of electrode array determined by ratio (a/b)

Next theorizations can be done from the point of view how looks ratio (a/b). We distinguish two all different extremal cases. The first is when it is a circular electrode array, the second holds for a stretched elliptical electrode array. The second one can tend to the electrode pad of Proximity Log.

For a = b you have (a/b) =1what presents rather circular annulus, whereas, for a ‹‹ b is (a/b) →0 and it is much stretched elliptical annulus. Let's return again to formulas derived for narrow and wide insulators.

For narrow insulator between current and potential electrodes the current one fills almost all inner surface of annulus. The potential annulus is wide. Formulas (43) and (44) are:

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\text{, and}\quad F_2 = +2\times\left(\frac{2}{\pi}\right).$$

The wide insulator reflects situation when the current electrode is very small tending to the point/abscissa. The potential annulus remains again wide. Formulas (45) and (46) are these:

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{\pi}{2}\right)\times\left(\frac{a}{b}\right)=2\times\left(\frac{a}{b}\right)\text{, and}\quad F_2 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{\pi}{2}\right)=2\,.$$

The first extremal case holds for circular electrode array. In such case you have to imply condition that (a/b) =1.

Circular electrode array; (a/b) =1.

For narrow insulator holds the following expressions:

$F_1 = +2\times\left(\frac{2}{\pi}\right)$, and $F_2 = +2\times\left(\frac{2}{\pi}\right)$. For their sum you receive that $F_1 + F_2 = +4\times\left(\frac{2}{\pi}\right)$.

For wide insulator you will get these formulas:

$F_1 = +2$, and $F_2 = +2$. The sum is following: $F_1 + F_2 = +4$.

The sum of functions F_1 and F_2 is nonzero in both cases. For the narrow insulator is even lower than for the wide one. For an ideal array should be respected both conditions simultaneously. Such ideal array is the pointed current electrode fills almost all the inner surface of the potential electrode formed as a wide annulus.

The second extremal case tends to an infinitely-long linear electrode array. Here holds condition that (a/b) →0.

Stretched elliptical electrode array (an elliptical electrode system with high eccentricity); (a/b) →0.

For narrow insulator you have these formulas:

$F_1 = 0$, and $F_2 = +2\times\left(\frac{2}{\pi}\right)$. Their sum is following: $F_1 + F_2 = +2\times\left(\frac{2}{\pi}\right)$.

For wide insulator you obtain such formulas:

$F_1 = 0$, and $F_2 = 2$. For their sum holds that $F_1 + F_2 = 2$.

The sum of functions F_1 and F_2 is again nonzero in both cases, but value is half in comparison to circular electrode array. Conditions for narrow and wide insulators must be again respected simultaneously. An ideal electrode array is the thin and much stretched elliptical current electrode fills almost inner surface of the potential electrode having form of wide and stretched elliptical annulus. The current electrode can even look as a thin metallic abscissa. Such electrode array can be replaced with rectangular electrodes being easier for manufacturing.

6 Control – deduction of formula for circular electrodes

We apply conditions that B = A and b = a. Then we receive this formula:

$$F_1 + F_2 = +4\times\left(\frac{2}{\pi}\right)\times\left[\mathrm{E}\left\{\frac{A}{a}\right\} - \left(\frac{H}{a}+1\right)^{-1}\times\mathrm{E}\left\{\left(\frac{A}{a}\right)\times\left(\frac{H}{a}+1\right)^{-1}\right\}\right]. \qquad (49)$$

Now, the variables denoted as A, a, present radiuses of circles. We have to use identities: a = (d/2) and A = (D/2) where **d** and **D** are diameters.

$$\left(\frac{k}{d}\right)\times 2 = \frac{2\pi}{F_1 + F_2}.$$

The constant 2 must be transposed into denominator of the fracture being on the right side of equation.

$$F = 2\times\left(F_1 + F_2\right) = +8\times\left(\frac{2}{\pi}\right)\times\left[\ \mathrm{E}\left\{\frac{D}{d}\right\} - \left(\frac{2H}{d}+1\right)^{-1}\times\mathrm{E}\left\{\left(\frac{D}{d}\right)\times\left(\frac{2H}{d}+1\right)^{-1}\right\}\ \right]. \tag{50}$$

where D…the diameter of the circular current electrode [m] and

d…the inner diameter of the potential current annulus [m].

The formula holds for D < d. This is formula derived for the circular electrode array. It is partial case of the elliptical one. As I made, for my own control, the direct way of derivation of final formula for circular electrodes too, I can say the formula is derived rightly. Consequence of that is both formulas for circular and elliptical electrodes are rightly derived.

7 The special case of the common potential and current electrodes; A ≡ M

In such case holds that U = 0. If two electrodes are identical then their potentials are identical too. It results in that difference between both potentials presenting voltage U equals to zero. An analytical way is following. You have to implement conditions into formulas (37) and (38) that: a = A and b = B; further, H = 0. In such case it holds that:

$$F_1 = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[\ \mathrm{E}\{1\} - \mathrm{E}\{1\}\ \right] = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[1-1\right] = 0, \text{ and } \quad F_2 = +2\times\left(\frac{2}{\pi}\right)\times\left[\ \mathrm{E}\{1\} - \mathrm{E}\{1\}\ \right] = +2\times\left(\frac{2}{\pi}\right)\times\left(\frac{a}{b}\right)\times\left[1-1\right] = 0.$$

As $F_1 = 0$ and $F_2 = 0$ too consequence of that is $(F_1 + F_2) = 0$; constant k is equal to infinity and $k_{AM}^{-1} = 0$. So in such case like that holds again it is a thing of geometry, in the first instance zero distance between both electrodes; result of that is U = 0. It is for both elliptical and circular electrodes. This property can bet easier counting of the main constant of Microlaterolog.

8 Conclusions

This analysis of the derived formulas confirms the following conclusions:

- The derived constant for elliptical electrodes can be used for counting of partial constants k_{AM} and k_{AN}; characteristics of the electrode system are well-measurable and so it is possible constants exactly to count.
- It is possible to get the final formula for the circular electrodes too, thanks to implication of condition equality for both half-axes ellipses.
- The surface of the potential electrode ought to be as big as possible, whereas, the surface of the current electrode as small as possible, but simultaneously covering almost all the inner surface of potential annulus. Between the current and potential electrodes is only thin but also functional insulator. The voltage will be nonzero and high.

- If it is that holds that A ≡ M, the voltage of the annulus electrode is zero and the partial constant tends to infinity. Consequence of that is the reciprocal value of the partial constant is equal to zero; $k_{AM}^{-1} = 0$; the effect is independent on the electrode shape.

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