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THEORY AND INTERPRETATION OF GAMMA-RAY LOGGING IN OIL/GAS BOREHOLES

TEORIE A INTERPRETACE GAMMA KAROTÁŽE VE VRTECH NA UHLOVODÍKY

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Abstract

This paper presents theory and interpretation of Gamma-Ray Logs in oil/gas boreholes. The crucial point is mathematical proceedings of the characteristic function that presents mathematical model making possible depict how well-logging curve will be look for different technical and geological characteristics of tool, borehole and bed. It is about modelling by two processes: emission of gamma-photons and their following absorption by rocks and mud; both are acting simultaneously. The result of that makes possible creation of the highest possible saturation by gamma-photons when both above processes are in coincidence. In such case the deflections registered are highest of all.

In this paper there is discussed, too, calibration with the help of standard shaped like coaxial cylinder manufactured from depleted uranium. It makes possible to use outer and inner standards, because characteristic the apparent activity is dimensionless. The last chapter is devoted to interpretation of shaliness in sedimentary rocks.

Abstrakt

Tato práce představuje teorii a interpretaci gama karotáže, jak se provádí ve vrtech na uhlovodíky. Klíčovým bodem je matematické odvození charakteristické funkce, která představuje matematický model umožňující zobrazit, jak karotážní křivka bude vypadat pro různé charakteristiky sondy, vrtu a vrstvy. Jde o modelování dvěma procesy: emisí gama-fotonů a jejich následnou absorpcí horninami a výplachem; oba působí současně. Výsledkem toho působení je možný vznik nejvyššího možného nasycení gama-fotony, kdy oba zmíněné procesy jsou v rovnováze. V takovém případě výchylky, které jsou registrovány, jsou nejvyšší.

V práci je také diskutována kalibrace s využitím standardu ve tvaru koaxiálního válce vyrobenému z ochuzeného uranu. Umožňuje používat vnější a vnitřní standardy, protože charakteristika zdánlivá specifická aktivita je bezrozměrná. Poslední kapitola je věnovaná interpretaci jílovitosti sedimentárních hornin.

Keywords
measuring of the natural gamma activity in boreholes

Klíčová slova
měření přirozené gama-aktivity ve vrtech

1 Introduction

This method belongs to passive ones of nuclear well-logging. Here are those radioactive elements important to the natural surroundings: ^{235}U, ^{238}U, ^{226}Ra, ^{232}Th and ^{40}K. Registration of gamma-rays in the borehole is made like either spectral or integral ones. If it is an integral registration, the radiation of various radioactive elements shall be calibrated like a radiation equivalent of the only element.

In this paper it is ^{238}U, because in the hydrocarbon prospection you have to engage in an integral registration. If you interpret factor of shaliness, you will take into account summary radiation of all radioactive elements being on the surface of shales. Although this way of registration has lower position than spectral registration that carries much more information about bed, it does not mean that is not important. In hydrocarbon prospection it has big significance. The spectral registration makes possible to do detailed measurements, you can differ siltstones and claystones, however, are rather slow. Therefore, they have lower significance for oil and gas prospection.

The unit of radiation can be [μR/h], even if it is not allowed unit yet; it is occasionally used yet because of geologists who need to make a correlation between new and older boreholes. Nevertheless, this paper offers like unit [μg ^{238}U-eq/t]. More in detail it will be explained in next chapters.

From position of geology, the Gamma-Ray Log is important for lithological differentiation of rocks. In crystalline rocks minimal deflections have basic and ultrabasic rocks; maximal deflections are for light alkaline rocks containing potassium. In the sediments the lowest deflections have biogenic sediments having no potassium as halite, anhydrite, gypsum and coal are, further, chemically-pure limestones and sandstones. The middle deflections are for marlstones and the highest ones are for claystones and shales. If in rocks is not an anomalous concentration of the uran-thorium minerals, in case of the sand-shale borehole section, you can determine after relative deflections of the Gamma-Ray Log shaliness of rocks having close relation to permeability. The relation presents indirect proportion. The gamma-Ray logs you can compare to SP logs, alternatively to Magnetic logs, when it is in the crystalline rocks.

Investigation of Gamma Ray Logs was taking place after Second World War across whole world, mainly in Soviet Union and United States. Both great powers needed uranium not only for producing nuclear weapons, but too for nuclear power stations. Uranium prospection was much strengthened. Russians were more focused on theory and scientific interpretation, Americans preferred sophisticated digital technique and standardization of recorded data.

From Russian studies it was a lot of authors, however, it was FILIPOV (1973) who laid really foundation stone of scientific theory of Gamma Ray Log. As for standardization Russians at the beginning preferred an integral registration to spectral one. The logs were calibrated in [or/h] and as a source was used ^{226}Ra.

In US it was in the first place Schlumberger Company which generated digital well-logging equipment. This made possible automatic registering and sophistic interpretation. Records of Gamma Ray Log needed to be exactly calibrated. It all directed to building first calibration pit. Standards were the concrete blocks having various activity of ^{226}Ra. As you see, also here the first was an integral registration. However later there was built the calibration pit for spectral registration. As the standards were used again concrete blocks with various volumes of these elements: ^{238}U, ^{232}Th and ^{40}K. This type of calibration pits were improved and enlarged over the world. How such calibration pit looked you can see in PIRSON (1966).

Czech Geophysics tried to keep step with the world development. It was MATOLÍN, who studied news of theory and all information about standardization. You can find that in works MATOLÍN (1987), MATOLÍN and DĚDÁČEK (1971), further MATOLÍN, et al. (2010) and MATOLÍN, KŘESŤAN, and STOJE (2011). Simultaneously there was studied development of digital well-logging equipment. The first digital well-logging equipment was constructed thanks to KŘESŤAN, J. and KŘESŤAN, P. The equipment was manufactured from domestic components. The sophisticated digital apparatus made possible new way of interpretation. KŘESŤAN, J. came with theory of elementary thin beds when each of data points has presented thin bed and its thickness was equal to the length of digital step. I have to say I accepted his idea too; the chapter 10 is made on this principle. More you can have in KŘESŤAN, STÁRA, and STUDNIČNÝ (1967).

The first calibration base, determined for both surface and borehole records, was built Bratkovice near Příbram. It was made for gamma-spectrometry for elements: ^{238}U, ^{232}Th and ^{40}K. In recent time this calibration base was transported into town Stráž pod Ralskem. More about you can find in MATOLÍN, KŘESŤAN, and STOJE (2011). What is important is how reflect interpreted data geological reality. Here I should like to point out the work STOJE, (1978). Results of interpretation in praxis you can find in KŘESŤAN, PEROUTKA (2007). The contribution to theory and interpretation of Gamma Ray Log can be found also in works of SCOTT et al. (1961), SCOTT (1963), CONAWAY and KILLEEN (1978), OLGAARD (1992), RIDER (1990), etc.

2 Vocabulary of some important characteristics used in this paper and their explanation

Here are maybe new, characteristics that are important for understanding of this paper. It is about the characteristic function, the specific activity of the nuclear source, the apparent activity of the nuclear source, radiation loss and the gamma-constant of the nuclear source.

2.1 The characteristic function

The characteristic function presents mathematical model making possible depict how well-logging curve will be look for different technical and geological characteristics of tool, borehole and bed. Modelling of the log curve is very important mainly in oil and gas prospection when thickness of bed, invasion zone, diameter of borehole make possible do an illustrative vision about bed.

Position of tool in the borehole has too its significance. The tool has two fixed extreme positions: the tool can be either centred in the axis of borehole, or pressed to the borehole wall. Within oil and gas prospection we often meet collectors of type sandwich. It presents alternating very thin beds of clean sands and shales/claystones. The sands can be highly saturated with hydrocarbons, however, when the tool is not centred, the log will present only tiny wavy line of shale. Those thin beds simply will not be there. Therefore is important to center the tool in axis of borehole. Figures Fig. 1, 2 and 3 present thin beds on condition h = d, when thickness of bed is equal to the diameter of borehole. Thin beds are most close to case for h << d; it is about h = 1 – 2 cm! Such beds are really very thin and deflections are much higher for centred tool than for pressed to tool.

2.2 The specific activity of the nuclear source

The specific activity of the nuclear source is denoted as $\sigma \equiv [kg \times kg^{-1}] = [g \times g^{-1}]$. As you can see, it is dimensionless characteristic. This characteristic presents amount of nuclear material in [kg] in 1kg of rocks. As amount of nuclear material is very small, there is used other unit being more suitable:

$\sigma \equiv [g\times g^{-1}] \times 10^{-6} = [t\times (t^{-1} \times 10^{-6})] = [(g\times 10^{6}) / (t \times 10^{6})] = 10^{6}\times [(g\times 10^{-6}) / t] = 10^{6}\times [\mu g\times t^{-1}] = [\mu g\ ^{238}U\text{-eq}/t] \times 10^{6}$.

For practice this unit is more comprehensible. It well expresses summary values of all various nuclear sources for integral registration. As it is dimensionless characteristic, has next advantage; for calibration you can use too inner calibrators, not only outer ones. Both types of calibrators can form the only system of calibration, when the inner calibrators can be tested with the help of outer calibrators on basis.

Names the specific and the apparent can be nothing special in the geophysics. You do know well the specific resistivity and the apparent resistivity used in well-logging and in the surface electric measurements.

2.3 The apparent activity of the nuclear source

The apparent activity of the nuclear source is just that characteristic that is registered. The characteristic is denoted similarly as $\sigma^* \equiv [kg\times kg^{-1}] = [g \times g^{-1}]$. Also here you can use unit, that: $\sigma^* \equiv [g\times g^{-1}] \times 10^{-6} = [\mu g\ ^{238}U\text{-eq}/t] \times 10^{6}$. It holds all what held for σ. Relations between both activities are different for the centred tool and for the pressed-to tool.

For the centred tool in next chapters holds:

$$\sigma^* = \frac{\pi}{2} \times \sigma \times \overline{D}_i \times f(\overline{z}, \overline{h},, \overline{D}_i, K).$$

Function $f(\overline{z}, \overline{h},, \overline{D}_i, K)$ is the characteristic function. Variables with line segment above are normalized to diameter of borehole. $\overline{z}$ is distance between the observer and the centre of bed, $\overline{h}$ is thickness of bed and K is the factor of radiation loss evoked by absorption with three factors: by density of environment ρ_b absorbing gamma-photons in the bed, the mass factor of gamma-absorption μ_m by the rock and by the borehole diameter d what presents radiation loess through distance.; factor K is dimensionless. Characteristic $\overline{D}_i$ presents the depth of invasion zone.

For the pressed-to tool in next chapters holds:

$$\sigma^* = \frac{\pi}{2} \times \sigma \times \overline{D}_i \times f(\overline{z}, \overline{h}, \overline{d}_s, \overline{D}_i, K).$$

Function $f(\bar{z}, \bar{h}, \bar{d}_s, \bar{D}_i, K)$ is again the characteristic function. Variables denoted as $\bar{z}$, $\bar{h}$ and K was explained yet. Here are two variables $\bar{d}_s$ and $\bar{D}_i$; both are normalized too. Symbol $\bar{d}_s$ is the diameter of tool and symbol $\bar{D}_i$ is again the depth of invasion zone.

2.4 The radiation loess

The radiation loess consists of two processes. The first is radiation loess evoked by transfer gamma-photons for certain distance. The second is radiation loess caused by absorption gamma-photons by material environment. It seems that the second process has in geology higher importance.

The integral exponential function of the first degree denoted as E_1 (M), presents summary distribution of the radiation loss. This characteristic is defined as follows:

$E_1(M) = \int_1^{\infty} \frac{e^{-M \times u}}{u} du$ where M is defined as $M = K \times \sqrt{(2\bar{z} \pm \bar{h})^2 + (2\bar{\Delta} + 1)^2}$.

$K = 0.5 \times \mu_m \times \rho_b \times d$.

Note, please, the function below the integral having form as $\{e^{-M \times u}/u\}$. Just this function accompanies too theory of the dielectric loess. The first partial function {1/u} is for transfer evoked by distance. The second partial function $\{e^{-M \times u}\}$ is for absorption evoked by environment. You can find it, for example, in DMITRIEV, et al. (1982).

Separation of both partial functions is well visible too on function denoted as M variables $\bar{\mathbf{z}}$, $\bar{\mathbf{h}}$ and $\bar{\mathbf{\Delta}}$ belong to loess evoked with transfer. All these are normalized to diameter d of borehole. Variables for K as ρ_b, μ_m and d, are, belong to loess for absorption. These are not normalized. What is interesting is position of variable d. This characteristic is contained both in radiation loess caused with transfer of gamma-photons and in radiation loess caused with absorption gamma-photons. Variable $\bar{\mathbf{\Delta}}$ is the eccentricity of tool what presents distance between the axis of tool and the axis of borehole.

2.5 The gamma-constant of the nuclear source

The gamma-constant of the nuclear source k presents experimentally-established value of the exposition power of gamma-radiation in [μR/h] for distance 1cm from gamma-source having quantity of radioisotope 1mCi = 3.7×10^7 Bq. It is older unit in present time it is 1 Becquerel [Bq]. If you work with [μR/h] you chance to it. This characteristic is too in older tables published for radioisotopes. You can find it, for example, in Tables (1964). ^{226}Ra has k = 8.40 μR/h, ^{60}Co has k = 13.25 μR/h et so on.

For the nuclear source having form as a point holds for counting of the exposition power of gamma-radiation in [μR/h] denoted as I, the basic formula:

$$I = \frac{k \times m}{r^2} \times e^{-\mu \times r} = k \times m \times \left(\frac{e^{-\mu \times r}}{r^2} \right)$$

As you see, the function in parentheses presents again radiation loess depending on distance r and absorption presented with characteristic μ what is the linear factor of gamma-absorption by the rock [m^{-1}].

3 Deduction of the basic formula needed for creation the characteristic function

We suppose we have the point nuclear source being in the centre of a rock sphere and acting on the surface of the virtual sphere. The change of radiation in such case expressed in [μR/h] is directed by formula:

$$dI = \frac{k \times m}{S} \times e^{-\mu \times r} \times \frac{dV}{V}, \qquad (1)$$

where k = the gamma-constant of the nuclear source [$\mu R/h \times m^2 \times kg^{-1}$],

m = the gamma-activity of the nuclear source [kg],

S = the spherical plane irradiated [m^2],

V= the rock bulk containing that nuclear source inside of the sphere [m^3], and

μ = the linear factor of gamma-absorption by the rock [m^{-1}].

The spherical plane is defined like that:

$$S = 4\pi \times r^2, \qquad (2)$$

where r = the distance between the bulk element denoted as dV and the detector [m]. It is radius of that virtual sphere.

The gamma-activity of the nuclear source is determined by next formula:

$$m = \sigma \times G, \qquad (3)$$

where σ = the specific activity of the nuclear source [$kg \times kg^{-1}$], and

G = the rock weight [kg].

Due to substitutions of formulas (2) and (3) into equation (1) you receive this expression:

$$dI = \frac{k \times \sigma \times G}{4\pi \times r^2} \times e^{-\mu \times r} \times \frac{dV}{V}. \qquad (4)$$

Formula (4) needs adjustment.

$$dI = \frac{1}{4\pi} \times \frac{\sigma \times k}{r^2} \times \frac{G}{V} \times e^{-\mu \times r} \times dV. \qquad (5)$$

The bulk density of rock denoted as ρ_b is presented like the following:

$$\rho_b = \frac{G}{V}. \qquad (6)$$

If you implement this relation to equation (5), you will attain that:

$$dI = \frac{1}{4\pi} \times \frac{\sigma \times k \times \rho_b}{r^2} \times e^{-\mu \times r} \times dV \, . \quad (7)$$

This equation I can adjust in this form:

$$\frac{\mu_m}{k} \times dI = \frac{1}{4\pi} \times \frac{\sigma \times \mu_m \times \rho_b}{r^2} \times e^{-\mu \times r} \times dV, \quad (8)$$

where μ_m = the mass factor of gamma-absorption by the rock [$m^2 \times kg^{-1}$].
Between the linear and mass factor there holds this relation:

$$\mu = \mu_m \times \rho_b \, . \quad (9)$$

If you substitute the above formula into equation (8), you will get that:

$$\frac{\mu_m}{k} \times dI = \frac{1}{4\pi} \times \sigma \times \frac{\mu_m \times \rho_b}{r^2} \times \exp\{ -\mu_m \times \rho_b \times r \} \times dV. \quad (10)$$

The change of the apparent activity denoted as $d\sigma^*$ is presented like this:

$$d\sigma^* = \frac{\mu_m}{k} \times dI \, , \quad (11)$$

where σ^* = the apparent activity of the nuclear source [$kg \times kg^{-1}$].
Formula (11) can be too expressed like this:

$$\sigma^* = \frac{\mu_m}{k} \times I \, , \quad (12)$$

where I = the exposition power of gamma-radiation in [μR/h].
The condition is to know constants k and μ_m. Both specify corresponding gamma-emitter.
The final form of the starting formula is as follows:

$$d\sigma^* = \frac{1}{4\pi} \times \sigma \times \frac{\mu_m \times \rho_b}{r^2} \times \exp\{ -\mu_m \times \rho_b \times r \} \times dV. \quad (13)$$

This is that basic formula needed for next derivation for derivation of the characteristic function. The formula holds on condition that it is **dry borehole** filled **with air**. In the rock there exist two processes acting against one another. The **first** is **emission** of gamma-photons and the **second** is **radiation loess** by **absorption** of same photons in the bed.

4 Creation of the characteristic function of Gamma-Ray Log

Modelling behaviour of active bed allows us to evaluate various influencing with geological and technical characteristics consequent measuring. It is possible to estimate various contour of thick or thin beds, consider curves between the centred and the pressed-on tools etc.

The characteristic function presents mathematical model making possible depict how well-logging curve will be look for different values factors of tool, borehole and bed. For derivation of this function we use formula (13). Accepting the cylindrical coordinates you will receive new equation as follows:

$$d\sigma^* = \frac{1}{4\pi} \times \sigma \times (\ \mu_m \times \rho_b) \times \frac{\rho}{\rho^2 + z^2} \times \exp\left\{ -\mu_m \times \rho_b \times \sqrt{\rho^2 + z^2}\ \right\} d\rho\, dz\, d\phi. \tag{14}$$

This formula needs an adjustment.

$$d\sigma^* = \frac{1}{4\pi} \times \sigma \times \left(\frac{d}{2}\right)^{-2} \times K \times \frac{\left(\frac{2\rho}{d}\right)}{\left(\frac{2\rho}{d}\right)^2 + \left(\frac{2z}{d}\right)^2} \times \exp\left\{ -K \times \sqrt{\left(\frac{2\rho}{d}\right)^2 + \left(\frac{2z}{d}\right)^2}\ \right\} d\rho\, dz\, d\varphi, \tag{15}$$

$$K = \frac{1}{2} \times \mu_m \times \rho_b \times d\ , \tag{16}$$

where K = the factor of radiation loss evoked by absorption of gamma-photons in the bed and with the borehole diameter together; this factor is dimensionless,

d = the borehole diameter [m],

ρ_b = the bulk density of rock [$kg\times m^{-3}$], and

μ_m = the mass factor of gamma-absorption by rock [$m^2 \times kg^{-1}$].

It is distinct that the loss of gamma-radiation in the dry borehole is bigger for increasing borehole diameter, increasing bulk density and for increasing mass factor, too. The boreholes having small diameter have small loss of gamma-radiation; they differ of those with big diameter.

By integration of formula (15) you attain this formula:

$$\sigma^* = \frac{1}{4\pi} \times \sigma \times \left(\frac{d}{2}\right)^{-2} \times K \times \int_0^{2\pi} \int_{z-h/2}^{z+h/2} \int_{r_0}^{\infty} \frac{\left(\frac{2\rho}{d}\right)}{\left(\frac{2\rho}{d}\right)^2 + \left(\frac{2z}{d}\right)^2} \times \exp\left\{ -K \times \sqrt{\left(\frac{2\rho}{d}\right)^2 + \left(\frac{2z}{d}\right)^2}\ \right\} d\rho\, dz\, d\varphi, \tag{17}$$

$$r_0 = \left(\frac{d}{2}\right) \times \left\{ (2\overline{\Delta}) \times \cos\varphi + \sqrt{1 - (2\overline{\Delta})^2 \times (\sin\varphi)^2}\ \right\}, \text{ and} \tag{18}$$

$$\overline{\Delta} = \frac{\Delta}{d}, \tag{19}$$

where Δ = the eccentricity of tool, i.e., the distance between the axis of tool and the axis of borehole [m],
σ^* = the apparent activity of rocks [kg × kg^{-1}], and
σ = the specific activity of rocks [kg × kg^{-1}].

In nature holds that adjacent beds have their own radioactivity σ_s as it is for studied bed. Modelling supposes the adjacent beds have zero level of radioactivity, so deflections are depicted from zero. You depict not σ^* but $\Delta\sigma^* = \sigma^* - \sigma_s$. Therefore for factual right further we will use $\Delta\sigma^*$ and not σ^*.

By implement of substitution t = (2ρ/d) and by application of next substitution that it holds that $w = \sqrt{t^2 + (2r/d)^2}$, you will get the new form of integration:

$$\Delta\sigma^* = \frac{1}{2\pi} \times \Delta\sigma \times \left(\frac{d}{2}\right)^{-1} \times K \times \int_0^{\pi} \int_{z-h/2}^{z+h/2} \int_{K \times \left[(2r_0/d)^2 + (2z/d)^2\right]^{1/2}}^{\infty} \frac{e^{-w}}{w}\, dw\, dz\, d\varphi. \tag{20}$$

Now, you can use Laplace transformation:

$$\frac{1}{w} = \int_0^{\infty} e^{-w \times u}\, du\ . \tag{21}$$

Thus, you obtain the next integration:

$$\Delta\sigma^* = \frac{1}{2\pi} \times \Delta\sigma \times \left(\frac{d}{2}\right)^{-1} \times K \times \int_0^{\infty} \int_0^{\pi} \int_{z-h/2}^{z+h/2} \int_A^{\infty} e^{-(1+u) \times w}\, dw\, dz\, d\varphi\, du. \tag{22}$$

$$A = K \times \sqrt{\left(\frac{2r_0}{d}\right)^2 + \left(\frac{2z}{d}\right)^2}\ . \tag{23}$$

Thanks to various substitutions and adjustments we make integrals for w, z and φ to the form suitable for solution with the help of the complex variable. Result of all that is an integral for variable u that is following:

$$\begin{aligned} \Delta\sigma^* = &+\Delta\sigma \times \left(\frac{\pi}{4}\right) \times K \times \left(2\overline{z} + \overline{h}\right) \times \exp\left\{-K \times \sqrt{\left(2\overline{z} + \overline{h}\right)^2 + \left(2\overline{\Delta} + 1\right)^2}\right\} \int_0^{\infty} (u+1)^{-1} \times \exp\left\{-K \times \sqrt{\left(2\overline{z} + \overline{h}\right)^2 + \left(2\overline{\Delta} + 1\right)^2} \times u\right\} du \\ &- \Delta\sigma \times \left(\frac{\pi}{4}\right) \times K \times \left(2\overline{z} - \overline{h}\right) \times \exp\left\{-K \times \sqrt{\left(2\overline{z} - \overline{h}\right)^2 + \left(2\overline{\Delta} + 1\right)^2}\right\} \int_0^{\infty} (u+1)^{-1} \times \exp\left\{-K \times \sqrt{\left(2\overline{z} - \overline{h}\right)^2 + \left(2\overline{\Delta} + 1\right)^2} \times u\right\} du\ . \end{aligned} \tag{24}$$

Now, you have to use substitution:

$$M = K \times \sqrt{\left(2\bar{z} \pm \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}\,. \tag{25}$$

Thus, you will be able to solve this integral:

$$e^{-M} \times \int_0^\infty \frac{e^{-M \times u}}{(u+1)}\,du\,.$$

With the help of the back Laplace transformation you will get this expression:

$$e^{-M} \times \int_0^\infty \frac{e^{-M \times u}}{(u+1)}\,du = e^{-M} \times \left\{-e^{M} \times \mathrm{E_i}(-M)\right\} = -\mathrm{E_i}(-M) = \mathrm{E_1}(M), \tag{26}$$

where $\mathrm{E_i}$ (M) = the integral exponential function, and
$\mathrm{E_1}$ (M) = the integral exponential function of the first degree. It presents distribution of the radiation loss.

Function $\mathrm{E_1}$ (M) is tabled and there holds the following relation:

$$\mathrm{E_1}(M) = \int_1^\infty \frac{e^{-M \times u}}{u}\,du = -\left[0.5772 + \ln M - \sum_{i=1}^{n} (-1)^{(i+1)} \times \frac{(M)^i}{i \times i\,!}\right]. \tag{27}$$

Thus, you shall receive that expected formula:

$$\Delta\sigma^* = \Delta\sigma \times \left(\frac{\pi}{4}\right) \times K \times \left\{\left(2\bar{z} + \bar{h}\right) \times \mathrm{E_1}\left\{K \times \sqrt{\left(2\bar{z} + \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}\right\} - \left(2\bar{z} - \bar{h}\right) \times \mathrm{E_1}\left\{K \times \sqrt{\left(2\bar{z} - \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}\right\}\right\}. \tag{28}$$

Formula (28) can be written like that:

$$\Delta\sigma^* = \frac{\pi}{2} \times \Delta\sigma \times \mathrm{f}\,(\bar{z}, \bar{h}, \bar{\Delta}, K)\,, \tag{29}$$

$$\mathrm{f}\,(\bar{z}, \bar{h}, \bar{\Delta}, K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\bar{z} + \bar{h}\right) \times \mathrm{E_1}\left\{K \times \sqrt{\left(2\bar{z} + \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}\right\} - \left(2\bar{z} - \bar{h}\right) \times \mathrm{E_1}\left\{K \times \sqrt{\left(2\bar{z} - \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}\right\}\right], \tag{30}$$

$$\bar{z} = \frac{z}{d}\,, \tag{31}$$

$$\bar{h} = \frac{h}{d}\,, \tag{32}$$

$$\bar{\Delta} = \frac{\Delta}{d} = \frac{1}{2} \times \left(1 - \bar{d_s}\right) = \frac{1}{2} \times \left(1 - \frac{d_s}{d}\right), \text{ and} \tag{33}$$

$$K = 0.5 \times \mu_m \times \rho_b \times d \,, \tag{34}$$

where $f(\bar{z}, \bar{h}, \bar{\Delta}, K)$ = the characteristic function for dry borehole having no invasion zone,

z = the distance between the point of observation and the centre of bed [m],

h = the thickness of bed [m],

Δ = eccentricity of tool [m], and

K = the factor of the radiation loss.

Both formulas (29) and (30) hold for dry borehole filled with air and on condition that there is no invasion zone, i.e., it holds that $(D_i/d) = 1$. Let's enlarge validity of formulas also for situation when the borehole is filled with mud and when the permeable radioactive bed has filtration.

5 Derivation of formulas for the borehole having mud and invasion zone

It was said yet we register the change of the apparent activity denoted by symbol σ*. It holds that:

$$\Delta\sigma^* = \sigma^* - \sigma_s \,, \tag{35}$$

where σ_s = the apparent activity of adjacent beds [kg × kg^{-1}].

We suppose adjacent beds have zero activity, i.e. $\sigma_s = 0$, whereas, the studied bed has high activity. In reality characteristic σ_s is non-zero, because has positive values, $\sigma_s > 0$. Depiction of modelling is made on relative differential level for $\sigma_s = 0$ when you can have both positive and negative values as it is in figures1, 2 and 3. In contrary depiction in absolute values has all data about curves only positive; however, size and shape of curves is remaining as it is for relative depiction. Very similar it is in the SP-potentials Log when you read data in [mV] from line of shales, whereas, shales have their own values in [mV] read from absolute zero.

Very frequent are beds of shales. They have highest values. Further, it is porous and permeable radioactive beds. Such bed can be modelled by mixture of sand grains and the grains of heavy radioactive minerals. All grains can be moreover consolidated with radioactive cement. Other model can be presented by coarse-grained sandstone having grains of potassium feldspar. If bedding water is not radioactive, even if the bed has an invasion zone, it is as $(D_i/d) = 1$. All other it is when the bed has bedding water being intensively saturated with radon and an invasion zone with non-radioactive mud filtrate. In case the homogeneous surroundings will change on inhomogeneous it will hold that characteristic $(D_i/d) > 1$.

Influence of mud can be made through the factor of radiation loss. It has two parts; for mud and for rock. It is time this all to attempt to express with the help of following formulas.

$$\Delta\sigma^* = \frac{\pi}{2} \times \Delta\sigma \times \bar{D}_i \times f(\bar{z}, \bar{h}, \bar{\Delta}, \bar{D}_i, K) \,, \tag{36}$$

$$\mathrm{f}\,(\bar{z},\bar{h},\bar{\Delta},\bar{D}_i,K)=\left(\frac{1}{2}\right)\times K\times\left[+\left(2\bar{z}+\bar{h}\right)\times \mathrm{E}_1\left\{K\times\sqrt{\left(2\bar{z}+\bar{h}\right)^2+\left[2\bar{\Delta}+\left(\bar{D}_i\right)^{-1}\right]^2}\right\}-\left(2\bar{z}-\bar{h}\right)\times \mathrm{E}_1\left\{K\times\sqrt{\left(2\bar{z}-\bar{h}\right)^2+\left[2\bar{\Delta}+\left(\bar{D}_i\right)^{-1}\right]^2}\right\}\right], \quad (37)$$

$$\bar{D}_i=\frac{D_i}{d}, \quad (38)$$

where D_i = the depth of invasion zone [m], and
d = the borehole diameter [m].

Next formulas hold for the factor of radiation loss.

$$K=K_r\times\bar{d}_s+K_f\times\left(1-\bar{d}_s\right), \quad (39)$$

$$K_r=\frac{1}{2}\times\mu_{mr}\times\rho_b\times d, \quad (40)$$

$$K_f=\frac{1}{2}\times\mu_{mf}\times\rho_f\times d, \text{ and} \quad (41)$$

$$\bar{d}_s=\frac{d_s}{d}, \quad (42)$$

where K_r = the factor of radiation loss in rock,
K_f = the factor of radiation loss in mud,
μ_{mr} = the mass factor of gamma-absorption in rock $[m^2 \times kg^{-1}]$,
μ_{mf} = the mass factor of gamma-absorption in mud $[m^2 \times kg^{-1}]$,
ρ_b = the bulk density of rock $[kg \times m^{-3}]$,
ρ_f = the density of mud fluid $[kg \times m^{-3}]$, and
d_s = the diameter of tool [m].

You should note, please, the term denoted as $\left(2\bar{z}\pm\bar{h}\right)$ presents effect of all characteristics acting **in vertical direction**, whereas, the term $\left(2\bar{\Delta}+1/\bar{D}_i\right)$ reflects effect of all characteristics **in horizontal direction**.

6 Gamma-Ray Log and SP-potential Log and their comparison

Relation between both well-logging methods is made, best of all, on their characteristic functions. Gamma-Ray Log is accompanied with two processes which are not separable each other. The rocks simultaneously emit and absorb gamma-photons; therefore it is not allowed to expect that absorption of gamma-photons does not exist. Nevertheless, you will see that there exists the state when is possible to be close to situation when process of absorption is negligible; then emission of gamma-photons is much higher than their absorption.

This stage is for K < K $^{(max)}$; more in chapter 9.1. Function E_1 {M} can be replaced by first term of series function f (M) when the mentioned term denoted as (1/M) is used as a rough approximation:

$$f(M) = 1 - \exp\left\{-\frac{1}{M}\right\} \text{ where } M = \left\{ K \times \sqrt{\left(2\bar{z} \pm \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2} \right\}.$$

$$E_1\left\{ K \times \sqrt{\left(2\bar{z} \pm \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2} \right\} \approx \left(1/M\right) = K^{-1} \times \left\{ \left(2\bar{z} \pm \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2 \right\}^{-\frac{1}{2}}. \quad (43)$$

Then you reach the following form of the characteristic function:

$$f\left(\bar{z}, \bar{h}, \bar{\Delta}, \bar{D}_i\right) = \frac{1}{2} \times \left[\frac{\left(2\bar{z} + \bar{h}\right)}{\sqrt{\left(2\bar{z} + \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2}} - \frac{\left(2\bar{z} - \bar{h}\right)}{\sqrt{\left(2\bar{z} - \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2}} \right]. \quad (44)$$

On condition that $\bar{D}_i = 1$, i.e., there is no invasion zone, formula (44) is changed on equation:

$$f\left(\bar{z}, \bar{h}, \bar{\Delta}\right) = \frac{1}{2} \times \left[\frac{\left(2\bar{z} + \bar{h}\right)}{\sqrt{\left(2\bar{z} + \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}} - \frac{\left(2\bar{z} - \bar{h}\right)}{\sqrt{\left(2\bar{z} - \bar{h}\right)^2 + \left(2\bar{\Delta} + 1\right)^2}} \right]. \quad (45)$$

This is the characteristic function of SP-potentials without invasion zone. The factor of radiation loss has gone away, an influence of gamma-absorption does not exist yet; it is why the state of desaturation for SP-potentials cannot be. For $\bar{\Delta} = 0$ you have the characteristic function published with DACHNOV (1967). So this is an explanation why SP-Log and Gamma-Ray Log are often so very similar to.

7 Analysis of the derived formulas

Here are analyse characteristics: $\bar{z}$, $\bar{h}$, $\bar{\Delta}$, $\bar{d}_s$ and $\bar{D}_i$. The characteristic K is analysed only marginally; very detailed analysis will be in the next chapters, in particular, for relation to the thickness of bed. Basic formulas are those denoted as (36), (37) and (39). Here they are.

$$\Delta\sigma^* = \frac{\pi}{2} \times \Delta\sigma \times \bar{D}_i \times f\left(\bar{z}, \bar{h}, \bar{\Delta}, \bar{D}_i, K\right),$$

$$f\left(\bar{z}, \bar{h}, \bar{\Delta}, \bar{D}_i, K\right) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\bar{z} + \bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z} + \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2} \right\} - \left(2\bar{z} - \bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z} - \bar{h}\right)^2 + \left[2\bar{\Delta} + \left(\bar{D}_i\right)^{-1}\right]^2} \right\} \right],$$

$$K = K_r \times \bar{d}_s + K_f \times \left(1 - \bar{d}_s\right).$$

7.1 Centred tool

In this case there holds condition that $\Delta = 0$ so is that:

$$\overline{\Delta} = 0. \tag{46}$$

Factor of radiation loss is directed by formula (39). The characteristic function attains on the above condition this form:

$$f(\overline{z}, \overline{h}, \overline{D}_i, K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\overline{z} + \overline{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\overline{z} + \overline{h}\right)^2 + \left(\overline{D}_i\right)^{-2}} \right\} - \left(2\overline{z} - \overline{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\overline{z} - \overline{h}\right)^2 + \left(\overline{D}_i\right)^{-2}} \right\} \right]. \tag{47}$$

If there is no invasion there, it holds that $\overline{D}_i = 1$ and you will get this equation:

$$f(\overline{z}, \overline{h}, , K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\overline{z} + \overline{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\overline{z} + \overline{h}\right)^2 + 1} \right\} - \left(2\overline{z} - \overline{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\overline{z} - \overline{h}\right)^2 + 1} \right\} \right]. \tag{48}$$

Formula (48) is used either for the bed having no invasion. Thanks to it there holds it that:

$$\Delta\sigma^* = \frac{\pi}{2} \times \Delta\sigma \times f(\overline{z}, \overline{h}, , K). \tag{49}$$

If there is an invasion, i.e., it holds that $\overline{D}_i > 1$, you have to use formula (47). Invasion zone can be between the water saturated with radon and mud filtrate. For infinitely-deep invasion $\overline{D}_i >> 1$ when it holds that $\overline{D}_i \to \infty$ you can implement the following condition:

$$\sqrt{\left(2\overline{z} \pm \overline{h}\right)^2 + \left(\overline{D}_i\right)^{-2}} \approx \sqrt{\left(2\overline{z} \pm \overline{h}\right)^2} \approx \left(2\overline{z} \pm \overline{h}\right).$$

In such case it is possible to write:

$$f(\overline{z}, \overline{h}, , K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\overline{z} + \overline{h}\right) \times E_1\left\{ K \times \left(2\overline{z} + \overline{h}\right) \right\} - \left(2\overline{z} - \overline{h}\right) \times E_1\left\{ K \times \left(2\overline{z} - \overline{h}\right) \right\} \right]. \tag{50}$$

7.2 Pressed-to tool

Here holds the following condition (51). Then you receive the characteristic function (52).

$$\overline{\Delta} = 0.5 \times \left(1 - \overline{d}_s\right). \tag{51}$$

$$f(\overline{z}, \overline{h}, \overline{d}_s, \overline{D}_i, K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\overline{z} + \overline{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\overline{z} + \overline{h}\right)^2 + \left[1 - \overline{d}_s + \left(\overline{D}_i\right)^{-1}\right]^2} \right\} - \left(2\overline{z} - \overline{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\overline{z} - \overline{h}\right)^2 + \left[1 - \overline{d}_s + \left(\overline{D}_i\right)^{-1}\right]^2} \right\} \right]. \tag{52}$$

For thick tool, $d_s \to d$, there is condition that $\overline{d}_s \to 1$. In such case it is again formula (46); the tool is **simultaneously** pressed-to and centred. And it holds that $K = K_r$.

For thin tool, $d_s << d$, it is possible to write that $\bar{d}_s \rightarrow 0$; then you obtain this formula:

$$f(\bar{z}, \bar{h}, \bar{D}_i, K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\bar{z}+\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z}+\bar{h}\right)^2 + \left[1 + \left(\bar{D}_i\right)^{-1}\right]^2} \right\} - \left(2\bar{z}-\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z}-\bar{h}\right)^2 + \left[1 + \left(\bar{D}_i\right)^{-1}\right]^2} \right\} \right]. \quad (53)$$

In such case it is clear that $K = K_f$.

If it holds that $\bar{D}_i = 1$ relation will acquire this form:

$$f(\bar{z}, \bar{h}, K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\bar{z}+\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z}+\bar{h}\right)^2 + 4} \right\} - \left(2\bar{z}-\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z}-\bar{h}\right)^2 + 4} \right\} \right]. \quad (54)$$

This formula is allowed to use for that case when thin tool is pressed-to the wall; the bed has no invasion.

If invasion zone exists there, you have condition that $\bar{D}_i > 1$ and there will hold again formula (53). On condition that the invasion zone is infinitely-deep $\bar{D}_i >> 1$ when it holds that $\bar{D}_i \rightarrow \infty$, you can implement the next condition:

$$\sqrt{\left(2\bar{z} \pm \bar{h}\right)^2 + \left[1 + \left(\bar{D}_i\right)^{-1}\right]^2} \approx \sqrt{\left(2\bar{z} \pm \bar{h}\right)^2 + 1}. \quad (55)$$

In this case the characteristic function is following:

$$f(\bar{z}, \bar{h}, K) = \left(\frac{1}{2}\right) \times K \times \left[+\left(2\bar{z}+\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z}+\bar{h}\right)^2 + 1} \right\} - \left(2\bar{z}-\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(2\bar{z}-\bar{h}\right)^2 + 1} \right\} \right].$$

In practice holds when you have $\bar{D}_i > 8$, it presents an infinity invasion zone.

8 Estimation of the loss extent of gamma-radiation

The factor of radiation loss is presented by formula (39). It has two components; for rocks and for mud. The component for rocks denoted as K_r is directed by equation (40) and it looks like that:

$$K_r = \frac{1}{2} \times \mu_{mr} \times \rho_b \times d.$$

The second component for mud is denoted as K_f. Its equation is formula (41):

$$K_f = \frac{1}{2} \times \mu_{mf} \times \rho_f \times d.$$

What is interesting is that both components reflect double way of the radiation loss; it is absorption evoked by characteristics $\bar{\mu}$ and ρ and absorption arisen by distance presented by characteristic d.

I should like to touch problem which will be more discussed in the next chapter. Between the radiation loss and thickness of bed there exist relation. Each thickness of bed has its maximal value of K denoted as $K^{(max)}$.

If emission of gamma-photons prevails over their absorption in the bed there will be formed accumulation of those photons. Most of them are concentrated in the centre of bed. The gamma-photons freely diffuse in the rock/mud and they present the gamma-photon gas. Emission and absorption of them is in unstable coincidence; the bed is incompletely saturated by them yet. This is the state denoted as the state of incomplete saturation by inequality $K < K^{(max)}$. Such case when it holds that K = 0 can never be, because no of characteristics μ_m, ρ and d are not equal to zero.

This state continues to the state of complete saturation by the gamma-photon gas. Emission and absorption are in stable coincidence for a certain value of the radiation loss; there exists maximal saturation with the photon gas. It holds that $K = K^{(max)}$. The highest accumulation of photons is just in the centre of bed there.

If emission of gamma-photons is lower than their absorption, you will observe constant deficit of free gamma-photons in the bed. The gamma-photons are concentrated near to boundaries of bed; in the centre of bed there is visible deficit of them. Such development tends to the state of desaturation; there are no free gamma-photons there. This is the state denoted like $K > K^{(max)}$.

You can note in adjacent beds presented by sands there are observed deficits of gamma-photons close to boundaries, whereas, the centre of sand has higher amount of photons. In shale as emitter where is high radiation the motion of gamma-photons runs from the centre of bed to both boundaries, however, in sand like accumulator there is low radiation and therefore the motion of photons runs in contrary from boundaries to the centre of bed.

Tab. 1 Data of the mass factor of gamma-absorption for rocks and materials; for ^{238}U

Material	**α**	**μ_m [$m^2 \times kg^{-1}$]**
Air	1	0.078
Water	1.1098	0.070
Sandstone	0.9980	0.078
Limestone	1.0000	0.078
Dolomite	0.9980	0.078
Anhydrite	0.9994	0.078

The mass factor of gamma-radiation depends on energy of radiation. This is published in FILIPOV (1973). This author published the ratio between the mass factor of gamma-absorption of various rocks and materials and the mass factor of air. Calculation is very simple after this formula:

$$\mu_m = \mu_{mv} \times \alpha^{-1}, \qquad (56)$$

where α = the ratio between the mass factor of rocks and air and

μ_{mv} = the mass factor of gamma-absorption for air [$m^2 \times kg^{-1}$].

The values of the ratio α and characteristic μ_m you will find out in adjusted Tab. 1 for ^{238}U.

For calculation you need to know the mass factor μ_{mv}. FILIPOV (1973) published for ^{226}Ra having radiation energy E_γ = 1.25 eV this value for μ_{mv} = 0.054 $cm^2 \times g^{-1}$. However, there is not included correction for bremsstrahlung there. Nevertheless, that is included yet in TABLES (1964) when for E_γ = 1.25 MeV it holds that μ_{mv} = 0.0263 $cm^2 \times g^{-1}$. The correction ratio denoted as γ between both above values is following: γ = 0.0540/0.0263 = 2.053. Factor γ is constant for all next calculations. It holds too for depleted uranium ^{238}U.

It is an integral registration calibrated with the help of depleted uranium. Energy of ^{238}U is that E_γ = 0.048 MeV. For that energy according to FILIPOV (1973), p.52 – tab., we get value that is denoted as μ_{mv} = 1.600 $cm^2 \times g^{-1}$. Its corrected value follows from the next formula:

$$\mu_{mv} = \mu^*_{mv} \times \gamma^{-1} . \tag{57}$$

If μ^*_{mv} = 0.1600 $m^2 \times kg^{-1}$ and γ = 2.053 there is valid that μ_{mv} = 0.078 $m^2 \times kg^{-1}$. Now, you have to return to formula (56). The results of calculation are depicted again in Tab. 1. You see that sedimentary rocks have the same mass factor; the only water has low value, however, very tiny. Let's estimate an interval for the characteristic K_r. It holds that: 0.070 $m^2 \times kg^{-1} \le \mu_{mr} \le$ 0.078 $m^2 \times kg^{-1}$, further holds that 1000 $kg \times m^{-3} \le \rho_b \le$ 2850 $kg \times m^{-3}$, and 0.1 m $\le d \le$ 0.25 m. For these data it follows that it holds: $0.35 \le K_r \le 2.78$. As the upper limit for the bulk density was taken the matrix density of dolomite.

Now, you can attempt to make estimation for the characteristic K_f; μ_{mf} = 0.070 $m^2 \times kg^{-1}$, 1000 $kg \times m^{-3} \le \rho_f \le$ 1400 $kg \times m^{-3}$, and 0.10m $\le d \le$ 0.25m. Thanks to these data it is possible to say that $0.350 \le K_f \le 1.225$. Note, please, that it is easily to grasp that lower diameter of borehole means, too, the lower loss of gamma-radiation. In such case it is observed the bigger effect of accumulation of gamma-photons in the bed.

Tab. 2 Data of the gamma-radiation loss when the characteristic function has its highest value; for state K = $K^{(max)}$

Centred tool	**$\bar{h}$**	**$K^{(max)}$**
	10	0.044
	5	0.087
	1	0.313
Pressed-to tool	**$\bar{h}$**	**$K^{(max)}$**
	10	0.043
	5	0.082
	1	0.198

9 Relation between bed thickness and maximal value of the factor of radiation loss

It was said that in the borehole there exist two processes, emission and absorption of gamma-photons, acting against one another. I studied relations (48) for the centred tool and (54) for thin tool pressed on the wall of borehole. If you implement that $\bar{z} = 0$ and condition (46) it holds:

$$f(\bar{h},, K) = \left(\frac{1}{2}\right) \times K \times \left(2\bar{h}\right) \times E_1\left\{ K \times \sqrt{\left(\bar{h}\right)^2 + 1} \right\} = \left(K \times \bar{h}\right) \times E_1\left\{ \sqrt{\left(K \times \bar{h}\right)^2 + K^2} \right\} . \tag{58}$$

If you lay substitution that x = (K $\times \bar{h}$), you will adjust formula (58) like that:

$$f(x) = x \times E_1\left\{ \sqrt{x^2 + K^2} \right\} . \tag{59}$$

Now, you have to express the first derivative of formula (59). It holds that:

$$\frac{d}{dx} f(x) = E_1\left\{ \sqrt{x^2 + K^2} \right\} + x \times \frac{d}{dx} E_1\left(\sqrt{x^2 + K^2} \right) \tag{60}$$

After differentiation of this relation it is possible to write:

$$\frac{d}{dx}\mathrm{f}\left(x\right)=\left[\ \mathrm{E}_1\left\{\sqrt{x^2+K^2}\right\}-e^{-\sqrt{x^2+K^2}}+K^2\times\frac{e^{-\sqrt{x^2+K^2}}}{\left(\sqrt{x^2+K^2}\right)^2}\ \right]=0. \tag{61}$$

Next substitution will be that $\mathrm{y}=\sqrt{\mathrm{x}^2+\mathrm{K}^2}$. It holds then:

$$\mathrm{E}_1\left\{y\right\}-e^{-y}+K^2\times\frac{e^{-y}}{y^2}=0. \tag{62}$$

Solution of such equation is this:

$$K^{(\max)}=\frac{0.442}{\sqrt{\left(\bar{h}\right)^2+1}}=\frac{\log e^{\left(1+0.05/e\right)}}{\sqrt{\left(\bar{h}\right)^2+1}}. \tag{63}$$

You will get it from condition that equation (62) coincides when it holds that y = 0.442. This value is on condition: $\bar{\mathrm{h}}\rightarrow 0$

Similarly, it will be for the pressed-to thin tool. Its equation is following:

$$K^{(\max)}=\frac{0.442}{\sqrt{\left(\bar{h}\right)^2+4}}=\frac{\log e^{\left(1+0.05/e\right)}}{\sqrt{\left(\bar{h}\right)^2+4}}. \tag{64}$$

Let's try to compare the values of $\mathrm{K}^{(\max)}$ for the centred tool and the pressed-to tool. Both are in Tab. 2. If you analyse formula (63), you will observe these relations: for $\bar{\mathrm{h}}\rightarrow\infty$ it holds that $\mathrm{K}^{(\max)}\rightarrow 0$, whereas, for $\bar{\mathrm{h}}\rightarrow 0$ it is valid that $\mathrm{K}^{(\max)}\rightarrow 0.442$. For formula (64) you will get similar relation: for $\bar{\mathrm{h}}\rightarrow\infty$ there holds it that $\mathrm{K}^{(\max)}\rightarrow 0$, whereas, for $\bar{\mathrm{h}}\rightarrow 0$ it holds that $\mathrm{K}^{(\max)}\rightarrow 0.221$.

Further, the double value of 0.442 determines the highest value of the characteristic function is in the centre of the thick bed. This value is 0.884. It means that for $\mathrm{K}=\mathrm{K}^{(\max)}$ there is valid this inequality:

$$\mathrm{f}\left(\bar{z},\bar{h},\bar{\Delta}\right)\leq 0.884. \tag{65}$$

This fact the characteristic function is not equal to one is thanks to coincidence between accumulation and absorption gamma-photons.

Note, please too, the before theoretically-derived intervals for K_r and K_f are high enough. Do not also forget that the radiation loss is computed after formula (39).

$$K=K_r\times\bar{d}_s+K_f\times\left(1-\bar{d}_s\right).$$

This formula makes possible that the calculated value will be rather in the lower values than those calculated. We can expect that real radiation loss will be somewhere in the interval: $0<\mathrm{K}<0.44$ for centred tool and $0<\mathrm{K}<0.22$ for pressed-to tool. It holds for $\mathrm{K}\leq\mathrm{K}^{(\max)}$, see Figs. 1 and 2; Fig. 3 is for $\mathrm{K}>\mathrm{K}^{(\max)}$.

10 Discussion over curves of the characteristic function

We have to distinguish three basic states which are characterized with relations like $K < K^{(max)}$, $K = K^{(max)}$ and $K > K^{(max)}$ are. This chapter discusses three basic states.

10.1 The state of incomplete saturation with gamma-photons

This state is directed by inequality that $K < K^{(max)}$. Emission of gamma-photons is higher than their absorption. In both the bed and adjacent rocks there is formed accumulation of gamma-photons, but, their quantity is not so high enough the bed to be completely saturated. The curves of the characteristic function are similar to those for the characteristic function of SP-potentials. There are no negative values there. Differences between the centred and pressed-to tools are tiny; visually, these curves are same, nevertheless, the centred tool is always a bit higher. Depiction is presented in Fig. 1. Curves are counted after formulas (48) and (54); indexes present input data.

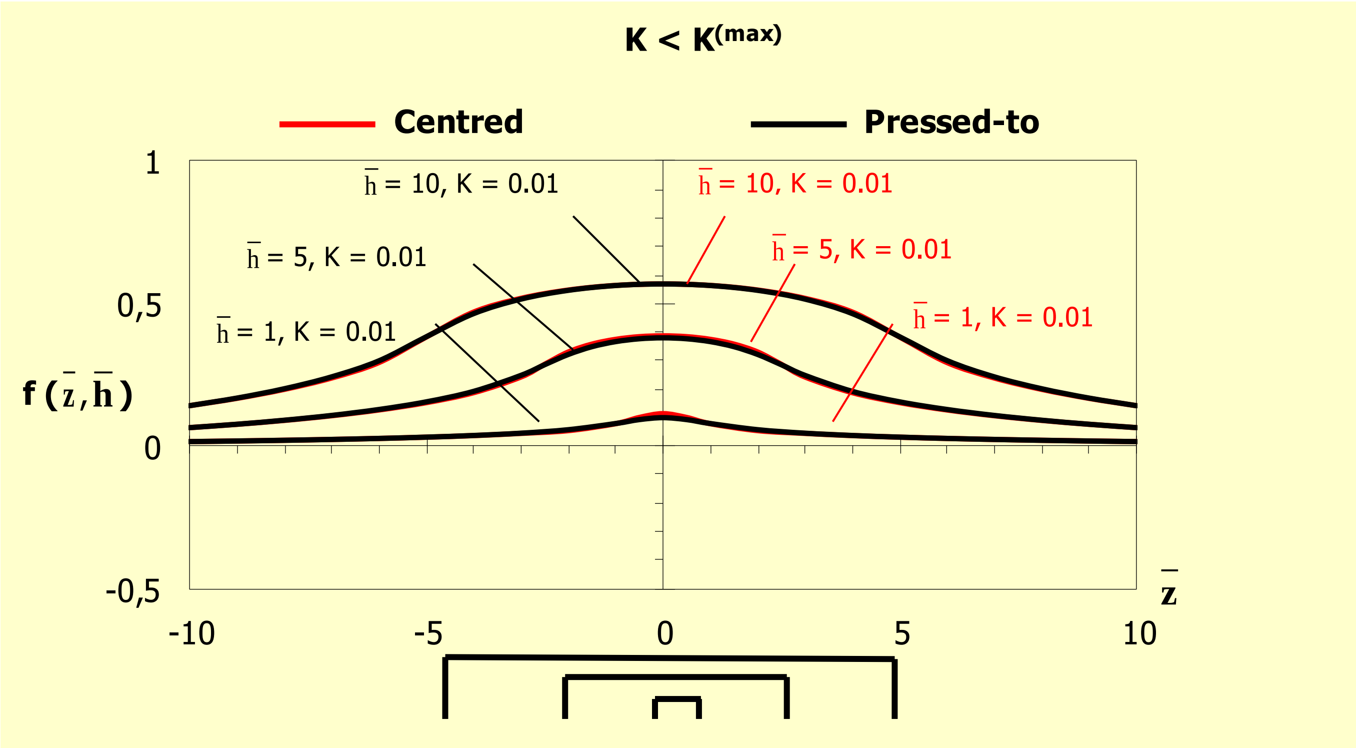

Fig. 1 Depiction positions of tool in the borehole for the state of incomplete saturation with gamma-photons

10.2 The state of complete saturation with gamma-photons

Here holds identity that $K = K^{(max)}$. It is characteristic for significant difference in amount of gamma-photons between adjacent rocks and an active bed. Emission and absorption are in coincidence, that is why complete saturation; the bed has highest constant accumulation of gamma-photons and is completely saturated by them. The deflections of curves are highest of all; it is because the active bed is sucking out gamma photons from adjacent rocks, mainly of borders. As a consequence of that is in adjacent rocks close to boundaries of bed there create two characteristic minima. Deflections of thick beds tend to the limit value equal to 0.884. Generally speaking, the centred tool has higher deflections than it is for the pressed-to tool. However, the differences for thick beds are almost negligible. The highest differences are for thin beds. All curves together have negative minima, but the characterizing crashing of curves for thick beds is not observed yet. Depiction is in Fig. 2 after formulas (48) and (54). It is important to compare Fig. 2 and Fig. 1; indexes of curves present input data.

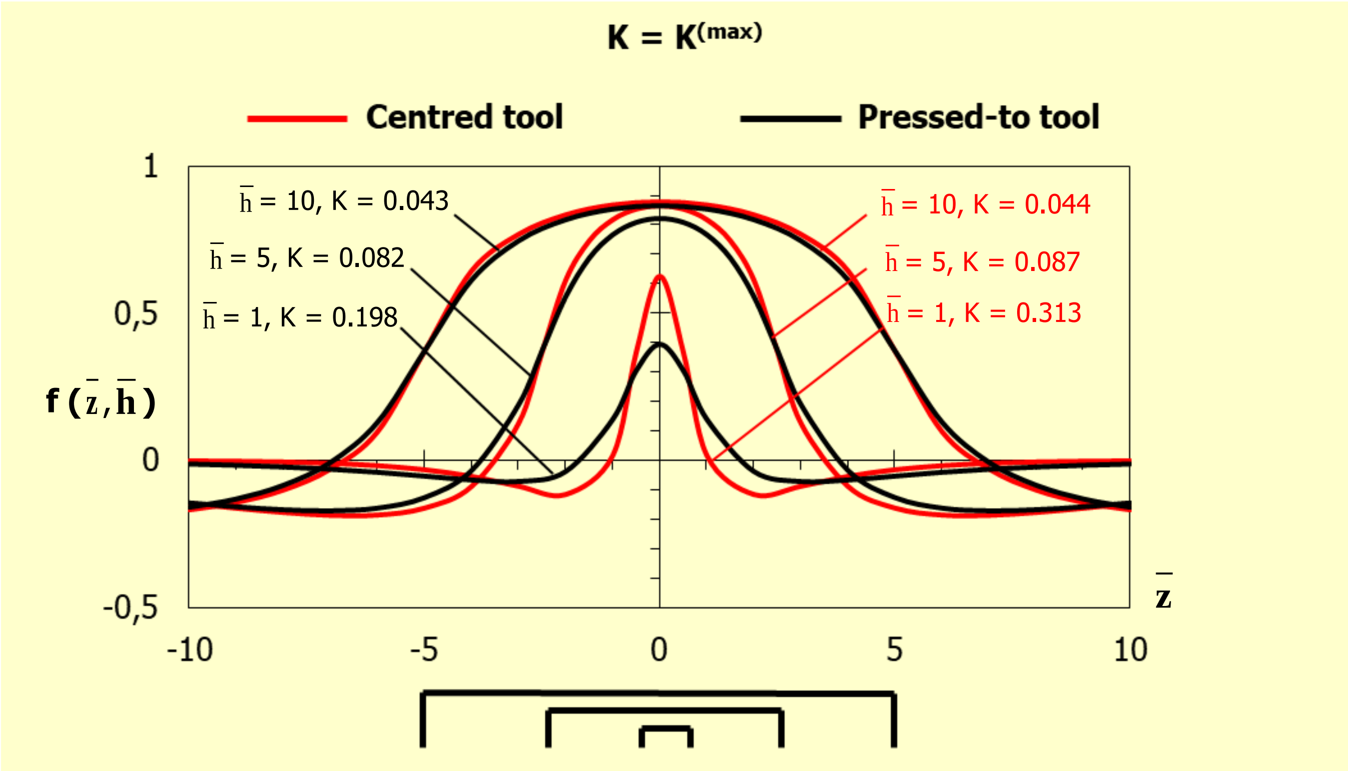

Fig. 2 Depiction positions of tool in the borehole for the state of complete saturation with gamma-photons

10.3 The state of desaturation evoked through absorption of gamma-photons

It is about Fig. 3, where emission of gamma-photons is lower than their absorption, because for both centred and pressed-to tools holds inequality that $K > K^{(max)}$. All maximal values of Tab. 2 are lower than K = 0.35. The emitted photons are rapidly absorbed thanks to auto-absorption gamma-photons evoked by higher density of consolidated active bed. Highest absorption is in the centre of bed; towards to boundaries is decreasing because the adjacent beds have low density. Therefore is formed a characteristic saddle, see Fig. 3. Accumulation of gamma-photons cannot be formed, because there is observed constant deficit of photons. Eventual state can be described as desaturation. The deflections are again lower than it was in the former case. The characteristic mark is crashing of curves for thick beds in the form of saddle; red and black curves in the case of thick beds are absolutely identical.

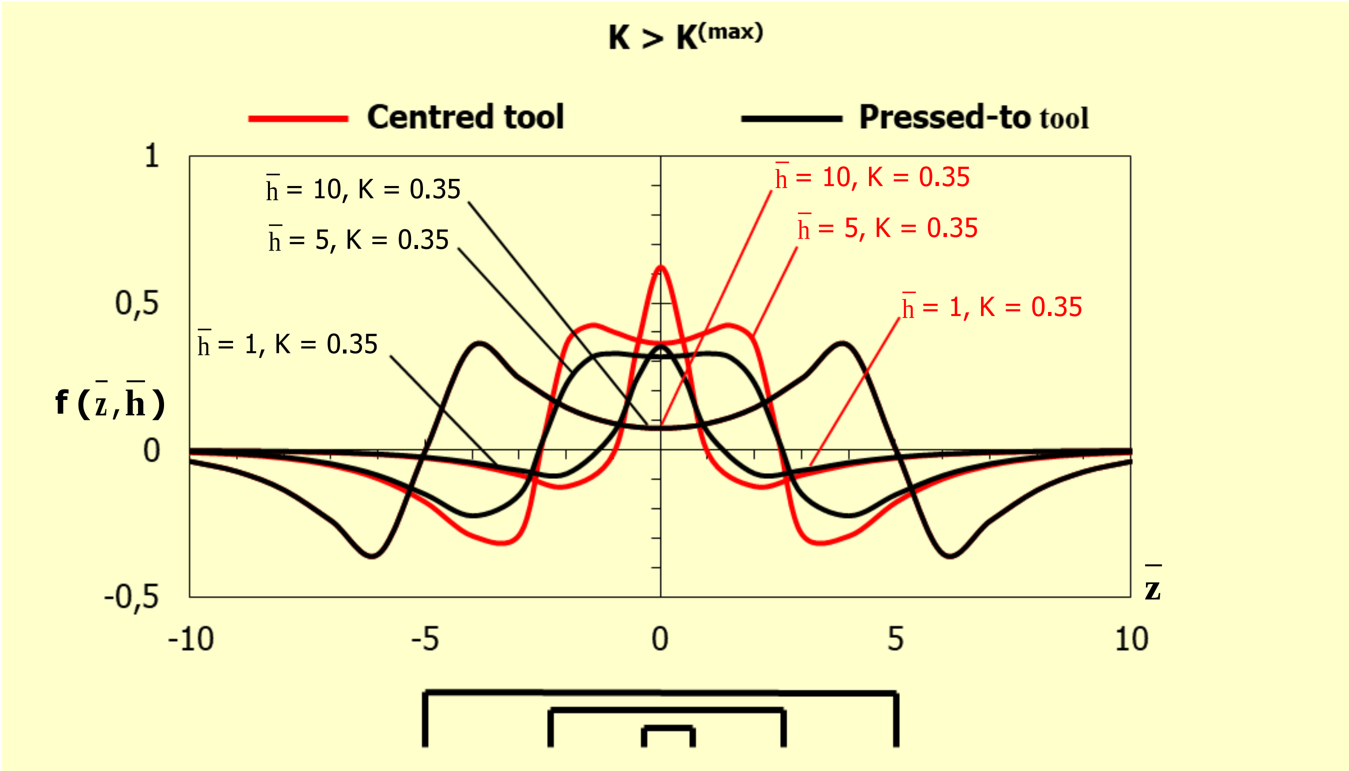

Fig. 3 Depiction positions of tool in the borehole for the state of desaturation through absorption of gamma-photons

The same, but in lower measure, is observed for the middle beds. Exception is the thin beds, because they have no crashing. What are common for all types of beds are well-visible negative minima, because radioactivity of adjacent beds is zero. The centred tool has again higher deflections than the pressed-to one; however, for the thick beds there are almost no differences there. What is possible to declare is in this domain there are well-observed thin beds, because mainly the thick beds thanks to saddle crashing are identified with difficulties. This all you find out in Fig. 3. Curves are counted after formula (48) and (54); indexes present input data.

11 Calibration of Gamma-Ray Log

We distinguish outer and inner standards. The outer are used more likely as indoor control on base, whereas the inner are advantageous for outdoor control for measurements in boreholes. The outer standards are used for controlling the inner standards; therefore their selected values are controlled in the accredited metrological laboratory. It needs to mention you can use too next a bit other unit for measurement and calibration. It is the mass activity of radionuclide [μBq/g], however in this theoretical study it is the apparent activity of rocks in [μg ^{238}U-eq/t], that remains for all this paper.

11.1 Outer standards

The outer standards are manufactured from depleted uranium ^{238}U. The measured characteristics σ^* and σ are dimensionless. It can be [kg × kg^{-1}], [g × g^{-1}] or [t × t^{-1}]; see formula (11). As Gamma-Ray Log presents absolute registering from absolute zero, because all types of rocks have their own values of the apparent activity, you have to use characteristics σ* and σ, but not Δ σ* and Δ σ needed for modelling.

If you have an outer standard manufactured in the form of coaxial cylinder having very thin wall and the material is depleted uranium, you will have the state of incomplete saturation, when it holds that $K < K^{(max)}$. The tool is centred and between tool and uranium cylinder is air gap.

In such case absorption of gamma-rays is not almost any and you are allowed to use formula (44) emerged from condition (43). Such standard imitates a uranium bed of finite thickness having no invasion that is bored through with certain diameter. As mud is air.

As the tool is centred there is valid that $\overline{\Delta} = 0$. Further holds that $\overline{D_i} = 1$. In this event you receive the following formulas:

$$\sigma^* = \frac{\pi}{2} \times \sigma \times f(\bar{z}, \bar{h}), \text{ and} \tag{66}$$

$$f(\bar{z}, \bar{h}) = \frac{1}{2} \times \left[\frac{\left(2\bar{z} + \bar{h}\right)}{\sqrt{\left(2\bar{z} + \bar{h}\right)^2 + 1}} - \frac{\left(2\bar{z} - \bar{h}\right)}{\sqrt{\left(2\bar{z} - \bar{h}\right)^2 + 1}} \right]. \tag{67}$$

Equation (67) has values what is very advantageous for calibration. As well it is too characteristic function for SP-potentials. Let´s attempt analyze next terms of equation (66). Pay an attention to characteristic being denoted as σ on the right side. Its dimension is following: [kg×kg^{-1}] = [g×g^{-1}]. It is not convenient unit; is too high. Therefore we use a lower one. It will be the unit having dimension [g× g^{-1}] ×10^{-6}. However, such unit needs a small adjustment.

We have relation: $[g\times g^{-1}]\times 10^{-6} = [t\times (t^{-1}\times 10^{-6})] = [(g\times 10^{6}) / (t\times 10^{6})] = 10^{6}\times [(g\times 10^{-6}) / t] = 10^{6}\times [\mu g\times t^{-1}] = [\mu g\ ^{238}U\text{-eq}/t]\times 10^{6}$. So it is advantageous to calibrate in [μg ^{238}U-eq/t]. This presents microgram-equivalent of depleted uranium per ton. In case of the outer standard formed as coaxial cylinder; the weight of standard is the same like the weight of depleted uranium ^{238}U. The weight divided by identical weight is 1; then it is clear that $\sigma = 1\mu g\ ^{238}U\text{-eq}/ t\times 10^{6}$. This characteristic is constant and it is fundament for next counting.

Now, you have all together. The cylindrical standard has its length denoted as h, further, its inner diameter denoted as d and it is possible to change the distance denoted as z between the centres of the standard and detector. These are the much needed characteristics. With the help of formulas (66) and (67) you receive the needed numeric characteristic σ^*. In this way you attain arbitrary amount of the point dyads needed for the calibration function presented as relationship $I= f(\sigma^*)$ where I is deflection of Gamma Ray Log.

If you have two standards having each various diameter denoted as d, it is possible to have very large range of the calibration points. In Tab. 3 there are data calculated due to formulas (66) and (67) as an illustration of counting, when holds that $\sigma = 1\mu g\ ^{238}U\text{-eq}/ t\times 10^{6}$.

11.2 Inner standards

As both characteristics σ^* and σ are dimensionless you can use calibration with the help of ratio of two voltages. It is advantageous when you do outdoor measurements in boreholes. The inner standards present two independent voltage sources denoted as U_1 and U_2 having accurately calibrated values in [mV]. Accuracy of both sources must be $\Delta U = \pm 0.1mV$, at least. By selection of values on both sources you reach arbitrary ratio which is conveyed to the scale of equipment. It is possible to do that, because the apparent activity of rocks is defined as dimensionless ratio of two independent factors; the ratio $(\sigma^*\times 10^6)$ is presented as [μg ^{238}U-eq/t] $\times 10^6$.

$$\left(\sigma^*\times 10^6\right) = \frac{U_1}{U_2}\times 10^6\ . \qquad (68)$$

The deflection on the recorder is denoted as l_0. It is presented by the following formula:

$$l_0 = \left(\sigma^*\times 10^6\right)/ n\ , \qquad (69)$$

where n = the step of linear scale expressed in $[\sigma^*\times 10^6 / 1cm]$.

11.3 Interpretation of the specific activity of rocks denoted as σ

For computation of the specific activity you can use formula (66). From that formula it results, that holds:

$$\sigma = \frac{\left(\frac{2}{\pi}\right)}{f(\bar{h},K)}\times\sigma^*, \qquad (70)$$

Where σ^*…the apparent activity of rocks [μg ^{238}U-eq/t],
σ …the specific activity of rocks [μg ^{238}U-eq/t], and

Tab. 3 The apparent activity of ^{238}U for various distances of calibration

$\bar{h}$	$\bar{z}$	$f(\bar{z},\bar{h})$	σ^* [μg ^{238}U-eq /t] $\times 10^6$
2	0	0.8944	1.404920
	0.5	0.8279	1.300462
	1	0.4851	0.761993
	1.5	0.1367	0.214728
	2	0.0460	0.072257

$f(\bar{h}, K)$ …the characteristic function after formula (47) when, $\bar{\Delta} = 0$, $\bar{z} = 0$ and $\bar{D}_i = 1$ having only dimensionless characteristics K and $\bar{h}$.

For computing both of them we need to know diameter of borehole in the concrete borehole point d. Interpretation is makes after depth points. Distance between two depth points presents thickness of the small bed. It is constant value. For general measurement can be h = 1m; then holds that $\bar{h}$ = (1/d). For detail measurement you need to have h = 0.1 m. In such case $\bar{h}$ = (0.1/d). Computing of the factor of radiation loss denoted as K needs not only characteristic d but too the bulk density ρ_b . Then you can get activity of rocks.

12 Interpretation of shaliness

The apparent activity of ^{238}U – it is that characteristic being well-usable for uranium prospecting. However, prospecting of hydrocarbons and fresh water carries other problem. It is the content of clay/shale in the rock. Elements ^{232}Th and ^{40}K are known to be adsorbed by surface of clay/shale. This fact offers opportunity to determine factor of shaliness in the sedimentary rocks. However, there must be made two premises. The first is the coefficient of adsorption of both mentioned elements ought to be stable for most of clays/shales, because only on this condition the change of the apparent activity reflects changes of the content of clay/shale in the rock. The second is the factor of shaliness is dependent on porosity of clay/shale. The soft and only tiny consolidated clays have lower shaliness than those well-consolidated shales. It is clear after figures 4 and 5.

We start from relation between the specific activity σ and double factor ΔI. Then the double index is defined like this:

$$\frac{\sigma - \sigma_{\min}}{\sigma_{\max} - \sigma_{\min}} = \frac{I - I_{\min}}{I_{\max} - I_{\min}} = \Delta I \; . \qquad (71)$$

You have find relationship between double factor and shaliness. In the literature there are published various experimental relations between ΔI and v_{sh} what is total shaliness containing wetness of shale w_j and the shaliness of the shale matrix v_j together as the only unit. Shale is bicomponent; the crucial role plays a role its porosity p_j. In Tab. 4 you can see what relations being between fundamental characteristics are and why are important for precious determination shaliness. All small change of the shale porosity caused with the terrastic pressure can evoke big change in permeability of rocks. Production of oil/gas can be strongly decreased.

Tab. 4 Fundamental characteristics reservoir engineering needed for oil and gas

	p	**m**	**v_{sh}**	**($p+m+v_{sh}$)**	**p_j**	**($1-p_j$)**	**w_j**	**v_j**	**($v_{sh} = w_j + v_j$)**
A	0.20	0.50	0.30	1	0.40	0.60	0.12	0.18	0.30
B	0.20	0.50	0.30	1	0.30	0.70	0.09	0.21	0.30

Between characteristics holds the following relations:

$$p + m + v_{sh} = 1, \tag{72}$$

where p … porosity of rock,

m … matrix of clean rock (clean grains) and

v_{sh} …total shaliness.

Next important relation is as follows:

$$v_{sh} = v_j + w_j, \tag{73}$$

where v_j … shaliness of the shale matrix and

w_j ….wetness of shale.

Relations between characteristics after formula (73) are these:

$$w_j = v_{sh} \times p_j. \tag{74}$$

$$v_j = v_{sh} \times \left(1 - p_j\right). \tag{75}$$

where p_j …porosity of shale,

$(1 - p_j)$….matrix of shale.

Tab. 4 carries two states of production hydrocarbons denoted as A and B. In both cases is total shaliness v_{sh} the same; $v_{sh} = 0.30$. However, shaliness of the shale matrix in case A is for $p_j = 0.40$ following; $v_j = 0.18$. When it is case B for $p_j = 0.30$ you get even $v_j = 0.21$. It presents considerable differences in values between characteristics v_{sh} and v_j. In practice it would present declining production of oil/gas, because permeability had been lower on account of increasing shaliness of the shale matrix by 3%. At the same moment the total shaliness v_{sh} would remain all the time on 30%. Note, please too, it holds always that $v_{sh} > v_j$.

Here are various formulas published by various authors for total shaliness.

$v_{sh} = 1.7 - [3.38 – (\Delta I + 0.7)^2]^{1/2}$ after Clavier, (76)

$v_{sh} = 0.083 \times (2^{3.7\Delta I} – 1)$ after Larionov for tertiary rocks, (77)

$v_{sh} = 0.33 \times (2^{2\Delta I} – 1)$ for older rocks (copied from album of nomograms Company Western Atlas International). (78)

Relationship $v_j = f(\Delta I)$ for shaliness of the shale matrix has form as follows:

$$v_j = \exp\left\{ - A \times \left[\ln\left(1/\Delta I\right) \right]^B \right\}, \tag{79}$$

where v_j … shaliness of the shale matrix.

The mentioned relation respects these conditions: for $\Delta I = 1$ it holds that $v_j = 1$ and for $\Delta I = 0$ it is valid that $v_j = 0$. Relation (79) can be adjusted too into form like that:

$$\ln\left(v_j^{-1}\right) = A \times \left[\ln\left(1/\Delta I\right) \right]^B \tag{80}$$

Association Dresser Atlas many years ago published relationship of types $v_j = f(\Delta I)$. It was made for soft tertiary rocks and for older well-consolidated rocks. Both are clay-rocks. Data are in Tab. 5. The curve denoted as 2 depicts the tertiary ones, whereas, the curve denoted as 1 presents the older consolidated ones. Both curves are depicted in Fig. 4. Formula (80) needs to be adjusted like a straight-line.

$$\ln\ln\left(\frac{1}{v_j}\right) = B \times \ln\ln\left(\frac{1}{\Delta I}\right) + \ln A. \qquad (81)$$

In Tab. 5 there are expressed coefficients A and B. The values of B are 0.9083 and 0.8933. Both are close to one other. It seems they reflect relation to **the weighted coefficient of surface adsorption**; its value is B = 0.90 ± 0.01. Coefficient B presents the weighted coefficient of the surface adsorption of radioactive elements in relation to surface of clay/shale. It is constant value for most of clay/shale. It says that 90 % of the surface of ones is attached by radioactive elements.

The coefficient A is related to porosity of clay/shale. The relation between them is following:

$$p_j = \left(1 - \frac{1}{A}\right). \qquad (82)$$

For A = 1.47346 you will get that $p_j = 0.32$ and this is valid for those older consolidated rocks presenting such rocks as claystones and shales are. If there are the tertiary rocks, what are soft and tiny consolidated clays, it holds that A = 2.08063 and you will attain the value that $p_j = 0.52$ what presents value on the start of consolidation. Both values of p_j well correspond to the observed reality. The correlation coefficients for curves 1 and 2 are 0.9996 and 0.9988 what is very close to 1. It means the relations are very tight

KONTA (1973) published the range of porosity for clays/shales yet. It confirms both limited data. After him the porosity clay/shale varies in the interval: $0.295 \le p_j \le 0.495$. If you suppose the interval $0.30 \le p_j \le 0.50$, the average porosity will be presently 0.40. It is that value needed for interpretation if you do not have any information about p_j. It is also possible to determine positions of clays/shales on the log of total porosity after next logs, to denote their intervals and to insert the curve of trend like function of the depth. Thus, you receive the continuous curve of p_j with the borehole depth for every depth points. So constructed curve of p_j can have a certain trend with depth; namely when the borehole is deep.

Tab. 5 Data of relations $v_j = f(\Delta I)$ after Fig. 4 presenting relations Dresser Atlas Association

v_j	ΔI_1	ΔI_2
0.05	0.10	0.18
0.10	0.20	0.30
0.15	0.27	0.40
0.20	0.34	0.47
0.25	0.40	0.53
0.30	0.45	0.59
0.35	0.50	0.64
0.40	0.55	0.68
0.45	0.60	0.73
0.50	0.65	0.76
0.55	0.69	0.79
0.60	0.73	0.82
0.65	0.77	0.85
0.70	0.81	0.87
0.75	0.84	0.90
0.80	0.88	0.92
0.85	0.92	0.94
0.90	0.94	0.96
0.95	0.97	0.98
Remarks	A = 1.47346 r = 0.9996 B = 0.90833	A = 2.08063 r = 0.9988 B = 0.89325

I constructed relationships $v_j = f(\Delta I, p_j)$ under conditions that B = 0.90 and $0.30 \le p_j \le 0.50$. The upper boundary $p_j = 0.50$ is very important, because for $p_j \le 0.50$ it is about rocks from soft clays up to well-consolidated shales, whereas the state when $p_j > 0.50$ it presents shifting ooze. For the mentioned relationships I used this formula:

$$v_j = \exp\left\{-\left(1-p_j\right)^{-1}\times\left[\ln\left(1/\Delta I\right)\right]^{0.9}\right\}. \qquad (83)$$

The result computed after this formula is in Fig. 5. Precise determination of shaliness has for hydrocarbon prospection an extraordinarily significance. Shaliness determines very strong how big will be an inflow of hydrocarbons/fresh water from bed. As we worked with the specific activity of rocks repaired of technical factors as the diameter of borehole and others are, you can use formulas:

$$\Delta\sigma = \frac{\sigma - \sigma_{\min}}{\sigma_{\max} - \sigma_{\min}} \text{ and} \qquad (84)$$

$$v_j = \exp\left\{-\left(1-p_j\right)^{-1}\times\left[\ln\left(1/\Delta\sigma\right)\right]^{0.9}\right\}. \qquad (85)$$

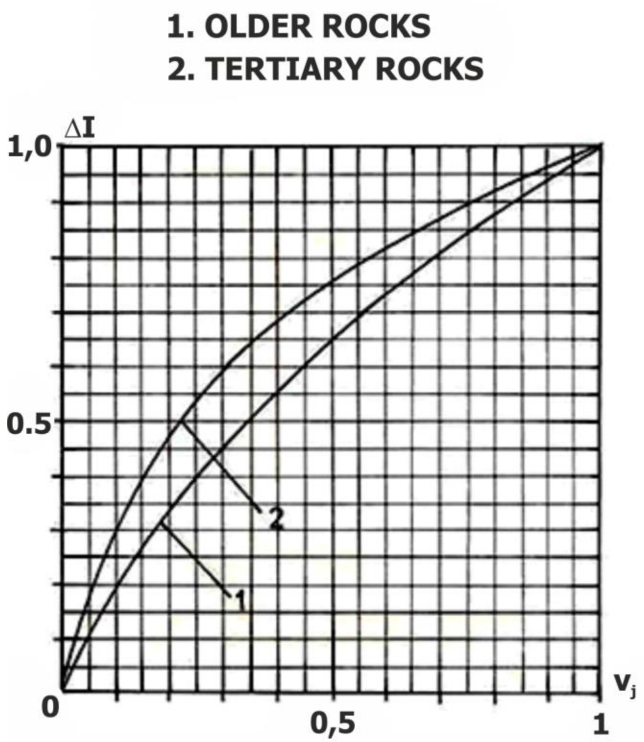

Fig. 4 Depiction of function $v_j = f(\Delta I)$ according to Dresser Atlas Association

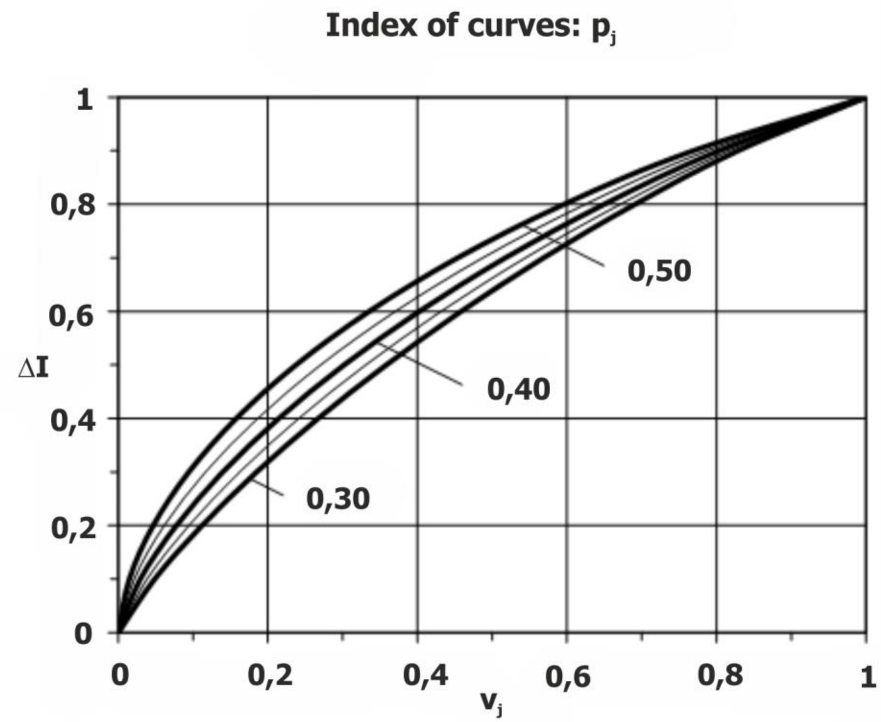

Fig. 5 Depiction of relation $v_j = f(\Delta I)$ for various data of p_j

13 Conclusions

As a result of the former analysis here are these conclusions:

- Gamma-Ray Log is universal method of well-logging both for determination of lithology for crystalline/sedimentary rocks and for evaluation of reservoir properties sediments thanks to shaliness.
- Concentration of uranium is possible to make either after bed by counting of anomaly plane of Gamma-Ray Log, or through counting of point by point from digital records. Both ways are used for U – Th deposits.
- Evaluation of the uranium concentration is possible to make too according to the apparent activity of ^{238}U.
- Calibration of Gamma-Ray Log after the apparent activity of ^{238}U can be made with the help of outer standards manufactured from depleted uranium. They have a form of coaxial cylinders.
- In reservoir prospection of oil/gas deposits is very important shaliness of sedimentary rocks, because can strongly influence permeability of rocks. Shaliness is determined with the help of double factor after deflections the characteristic measured.
- Interpretation of shaliness is also dependent on humidity of clay/shale that determines shaliness of the shale matrix. This characteristic influences significantly permeability of bed and is more accurate than total shaliness. Double information about shaliness is possible to be obtained by SP method, as well.
- An influence of technical and geological characteristics of the borehole and bed are is determined with the characteristic function of Gamma-Ray Log. The characteristic function makes possible mathematical modelling influenced with various characteristic as diameter of borehole, thickness of bed, depth of invasion zone and next ones are.

References

CONAWAY, J.G. and KILLEEN, P.G. Quantitative uranium determination from gamma-ray logs by application of digital time series analysis. *Geophysics*, 1978, Vol.43, pp.1204-1221.

DACHNOV ,V.N. *Električeskie i magnitnye metody issledovania skvazhin*. Nedra, 1967, 309s.

DMITRIEV, V.I. et al. Vyčislitelnyje matematika i technika v razvedocnoj geofyzike. *Handbook*, Moskva, Nedra, 1982, 218s.

FILIPOV, E.M. *Nuclear geophysics – Radiometry and radioisotopic geophysics with alpha-, beta- and gamma-radiators*, 1. vyd. Novosibirsk, Nauka, 1973, 513s.

KONTA, J. Imbibometrie. *Acta Universitatis Carolinae, Geologia*, Monography 1, Prague, 1973.

KŘESŤAN, J., STÁRA, S., STUDNIČNÝ, I. *O možnosti aplikace bezjádrového vrtání na ložisku Hamr u České Lípy*, Hamr u České Lípy, Dílčí závěrečná zpráva GPUP, 1967.

KŘESŤAN, P., PEROUTKA, J. Použití karotážních metod v rámci experimentu "Imobilizace kontaminantů in situ". *Sborník vědeckých prací Vysoké školy báňské – Technické univerzity Ostrava, řada stavební*, roč. 7, č. 2., 2007, s. 145 – 150: il.

MATOLÍN, M., DĚDÁČEK, K. *Studie vybudování cejchovací základny pro terénní gama spektrometry v ČSSR*. Zpráva FMPE a PřF UK, Praha, 1971.
MATOLÍN, M. *Stanovení technických parametrů cejchovací základny pro terénní gama spektrometry v Bratkovicích u Příbrami*. Zpráva PřF UK Praha a UP Liberec, Praha, 1987.
MATOLÍN, M., KŘESŤAN, P., STOJE, V., VESELÝ, P. Přestěhování kalibrační základny pro terénní spektrometry gama z Bratkovic do Stráže pod Ralskem, etapa1. *Sborník vědeckých prací Vysoké školy báňské – Technické univerzity Ostrava, řada stavebn*í, roč. 10, č. 2., 2010, s. 68 – 78: il.
MATOLÍN, M., KŘESŤAN, P., STOJE, V. Standardy pro terénní spektrometry gama ve Stráži pod Ralskem. *Journal EGRSE*, vol. XVIII, no. 3, 2011, p. 98 – 105, ISSN.
OLGAARD, P. L. A new approach to the interpretation of nuclear logs – *Geological Applications of Wireline Logs, II, - Geological Society*, *Special Publication* No 65, pp. 349-358, London,1992.
PIRSON, S.J. *Handbook of well log analysis*. Petroleum Engineering Department, The University of Texas, Edicion Revolucionaria, Instituto del Libro, Havana, 1966.
RIDER, M.H. Gamma-ray log shape used as a facies indicator: critical analysis of an oversimplified methodology – Geological Applications of Wireline Logs. *The Geological Society Special Publications*, London, 1990, pp.27-37.
RYŠAVÝ, F. Derivation of Gamma-Ray Exposure Velocity of the Ring Standard on The Tube Detector. *Journal EGRSE*, vol. V, no.2, 1998, p. 3 – 6, ISSN.
SCOTT, J.H. Computer analysis of gamma-ray logs. *Geophysics*, Vol.23, 1963, pp. 457-465.
SCOTT, J.H., DODD, D. H., DROULLARD, R. F. and MUDRA, P. J. Quantitative interpretation of gamma-ray logs. *Geophysics*, Vol.26, 1961, pp. 182-191.
STOJE, V. *Ověření automatické interpretace gama karotáže na fyzikálním modelu.* Dipl.práce PřF UK, Praha, 1978.
Jaderně chemické tabulky, SNTL, Prague, 1964 (in Czech).
ZELLER, E.J., DRESCHHOFF, G., HOLDOWAY, K. and SAUNDERS, D.F. Use of gamma-ray logs from wells drilled for petroleum to define uranium provinces in subsurface formations - Exploration for uranium ore deposit. *Proceedings of a symposium*, Vienna 1976.

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