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THE DISTRIBUTION MATRIX COEFFICIENTS OF THE CCBO METHOD

KOEFICIENTY DISTRIBUČNÍ MATICE METODY CCBO

Alice Petrlikova[1], Lubomir Stas [2], Petr Konicek[3]

Abstract

The Compact Conical Borehole Overcoring (CCBO) method is used to determine the in-situ stress state of rock masses. Due to the conical shape of the probe, an analytical solution for converting the measured strains into the corresponding stress tensor is not available, and a numerical model must therefore be applied. This model provides the distribution matrix, which links the observed strains to the stress tensor. The method was first developed in Japan in the early 1990s, where a semi-analytical procedure was applied to calculate the coefficients of the distribution matrix. In this study, the coefficients are obtained both directly from a numerical model based on simulated strain responses and via subsequent regression analysis. The coefficient estimates are presented together with their standard errors to quantify the reliability of the regression analysis. Finally, these coefficients are compared with those obtained using the original Japanese methodology.

Abstrakt

Metoda CCBO (Compact Conical Borehole Overcoring) se používá k určování stavu napjatosti in-situ v horninovém masivu. Vzhledem ke kuželovitému tvaru sondy dosud nejsou známy analytické vztahy pro přepočet mezi naměřenými přetvořeními a hledaným tenzorem napjatosti, a proto je nutné použít numerický model, na jehož základě je možné sestavit distribuční matici, která převádí naměřená přetvoření na složky tenzoru napjatosti. Metoda byla poprvé vyvinuta v Japonsku na počátku 90. let, kde byl ke stanovení distribuční matice použit semi-analytický přístup. V této studii jsou koeficienty získány jak přímo z numerického modelu na základě simulovaných odezev přetvoření, tak také prostřednictvím regresní analýzy. Spolu s odhady koeficientů jsou uvedeny i jejich standardní chyby, aby bylo možné kvantifikovat spolehlivost výsledků regrese. Nakonec jsou tyto koeficienty porovnány s koeficienty získanými původním japonským přístupem.

Keywords

Stress tensor, stress state in the rock mass, CCBO method, distribution matrix, distribution coefficients

Klíčová slova

Tenzor napjatosti, napjatost v horninovém masivu, metoda CCBO, distribuční matice, distribuční koeficienty

1 Introduction

The physical principle underlying the Compact Conical Borehole Overcoring (CCBO) method is the release of the immediate vicinity of the probe, which had been subjected to the original primary stress, through the deformations induced by the overcoring process (Fig. 1). This method requires a numerical model that takes into account the overcoring of the probe under the original primary stress, in order to determine the necessary relationships between stress and strain (Knejzlík et al. 2008; Petrlíková, 2024; Sugawara & Obara 1999).

Implemented in Midas GTS NX 2023 (v1.1; MIDAS IT Co., Ltd.), the numerical model follows the superposition principle by applying unit stress states to the rock mass surrounding the probe. This model simulates the actual overcoring process, with the deformation response of the surrounding rock environment obtained via strain gauge readings. These responses are monitored under the action of individual superposition states and compiled into a matrix called a 'distribution matrix'.

The distribution matrix is an approximate solution to the constitutive relationship between stress and strain (Eq. 1), incorporating information about the stiffness of the material and the transformations between the orthogonal coordinate system and the specific "conical" coordinate system in which the strain gauges are directionally arranged on the conical probe.

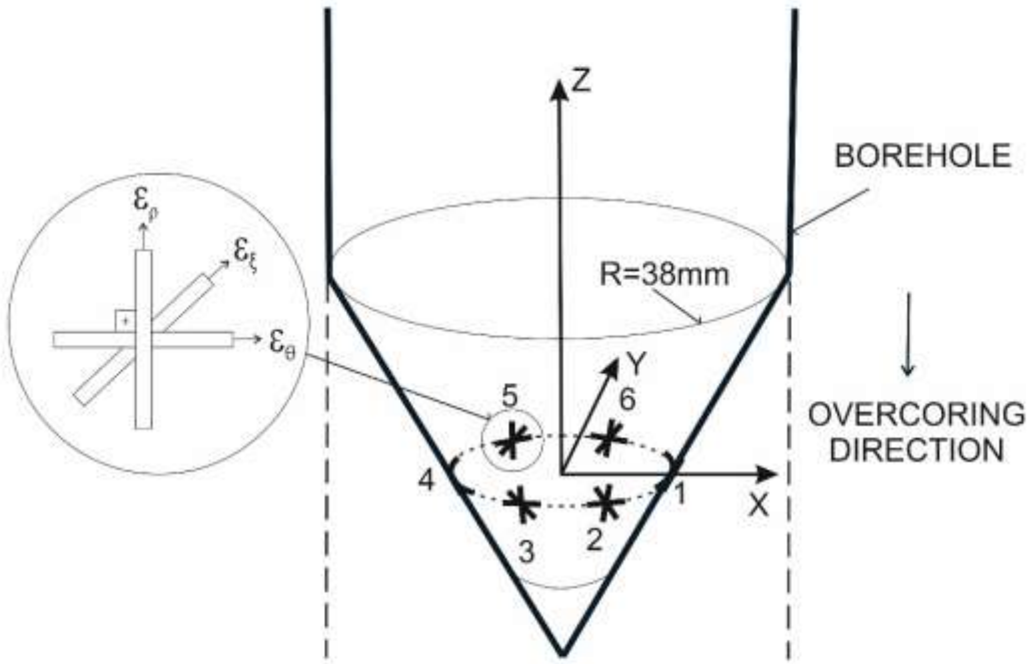

***Fig. 1** CCBO probe situated at the end of the borehole and then overcored (ε_{ij} – strain indicator in directions of individual strain gauges; i – position of the strain rosette around the circumference of the probe, i = 1–n, n = 6; j – direction of the individual strain gauge within the rosette (θ, ρ, or ξ). (Figure adapted and modified from Petrlíková, 2024)*

$$(\boldsymbol{\varepsilon}) = (\boldsymbol{D}) \cdot (\boldsymbol{\sigma}) \tag{1}$$

where
ε – column matrix of measured strains [–],
σ – column matrix of sought stress [MPa],
D – distribution matrix [GPa^{-1}].

In elaborated form, then:

$$\begin{pmatrix} \varepsilon_{\theta 1} \\ \varepsilon_{\rho 1} \\ \varepsilon_{\xi 1} \\ \vdots \\ \varepsilon_{\theta n} \\ \varepsilon_{\rho n} \\ \varepsilon_{\xi n} \end{pmatrix} = \begin{pmatrix} k_{1(\varepsilon_\theta,\sigma_a)} & k_{1(\varepsilon_\theta,\sigma_b)} & k_{1(\varepsilon_\theta,\sigma_c)} & k_{1(\varepsilon_\theta,\sigma_d)} & k_{1(\varepsilon_\theta,\sigma_e)} & k_{1(\varepsilon_\theta,\sigma_f)} \\ k_{1(\varepsilon_\rho,\sigma_a)} & k_{1(\varepsilon_\rho,\sigma_b)} & k_{1(\varepsilon_\rho,\sigma_c)} & k_{1(\varepsilon_\rho,\sigma_d)} & k_{1(\varepsilon_\rho,\sigma_e)} & k_{1(\varepsilon_\rho,\sigma_f)} \\ k_{1(\varepsilon_\xi,\sigma_a)} & k_{1(\varepsilon_\xi,\sigma_b)} & k_{1(\varepsilon_\xi,\sigma_c)} & k_{1(\varepsilon_\xi,\sigma_d)} & k_{1(\varepsilon_\xi,\sigma_e)} & k_{1(\varepsilon_\xi,\sigma_f)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{n(\varepsilon_\theta,\sigma_a)} & k_{n(\varepsilon_\theta,\sigma_b)} & k_{n(\varepsilon_\theta,\sigma_c)} & k_{n(\varepsilon_\theta,\sigma_4)} & k_{n(\varepsilon_\theta,\sigma_5)} & k_{n(\varepsilon_\theta,\sigma_6)} \\ k_{n(\varepsilon_\rho,\sigma_a)} & k_{n(\varepsilon_\rho,\sigma_b)} & k_{n(\varepsilon_\rho,\sigma_c)} & k_{n(\varepsilon_\rho,\sigma_4)} & k_{n(\varepsilon_\rho,\sigma_5)} & k_{n(\varepsilon_\rho,\sigma_6)} \\ k_{n(\varepsilon_\xi,\sigma_a)} & k_{n(\varepsilon_\xi,\sigma_b)} & k_{n(\varepsilon_\xi,\sigma_c)} & k_{n(\varepsilon_\xi,\sigma_4)} & k_{n(\varepsilon_\xi,\sigma_5)} & k_{n(\varepsilon_\xi,\sigma_6)} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{pmatrix} \tag{2}$$

where the individual coefficients of the matrix D are denoted by their azimuthal position around the probe (i = 1–n, n = 6) and characteristic orientation within the nest ρ, θ, ξ (see Fig. 1). The columns of matrix D vary depending on the superposition state from which the given coefficient originates (see Eq. 3).

$$(\boldsymbol{\sigma_a}, \boldsymbol{\sigma_b}, \boldsymbol{\sigma_c}, \boldsymbol{\sigma_d}, \boldsymbol{\sigma_e}, \boldsymbol{\sigma_f}) = -\boldsymbol{I_6} \tag{3}$$

where $-I_6$ denotes the negative identity matrix of order six. Its columns represent the superposition stress states, each defined by a single non-zero entry of −1MPa. The position of this entry follows directly the order of the stress tensor components introduced in Eq. 2.

The distribution matrix can be derived using two different approaches, both of which require the use of a numerical model. In the initial approach, the distribution matrix is constructed parametrically for varying values of Poisson's ratio. It is important to note that the applicability of this method is contingent upon the assumption of an isotropic rock medium. In this case, the stiffness is characterised by the Poisson’s ratio and Young’s modulus. This circumstance enables Young’s modulus to be factored out of the distribution matrix in Eqs. 1 and 2, so that the problem can be solved entirely at the “strain level”, with Young’s modulus reintroduced only at the final stage of the computational process. The distribution matrix is thus composed of coefficients that are not fixed numerical values but functions, which describe the dependence of the parametric coefficients on the position along the perimeter of the conical probe. The parametric coefficients, denoted as T, L, P (see Tab. 1 and 2), vary depending on the value of Poisson's ratio. The advantage of this approach is that once the distribution matrix has been established for specific values of Poisson's ratio, no further use of the numerical model is required.

The second approach involves determining the distribution matrix directly, with the numerical model defined by a specific stiffness of the rock medium. In this instance, the coefficients of the distribution matrix, corresponding to the individual strain gauges (both nests and orientations), are directly incorporated into the computational process. In this context, the coefficients in the distribution matrix are expressed as values directly read from the numerical model, not as functional relationships. This approach is applied when evaluating stresses in a rock mass under the assumption of material anisotropy (Aminzadeh et al. 2022) and is here referred to as the direct distribution matrix method.

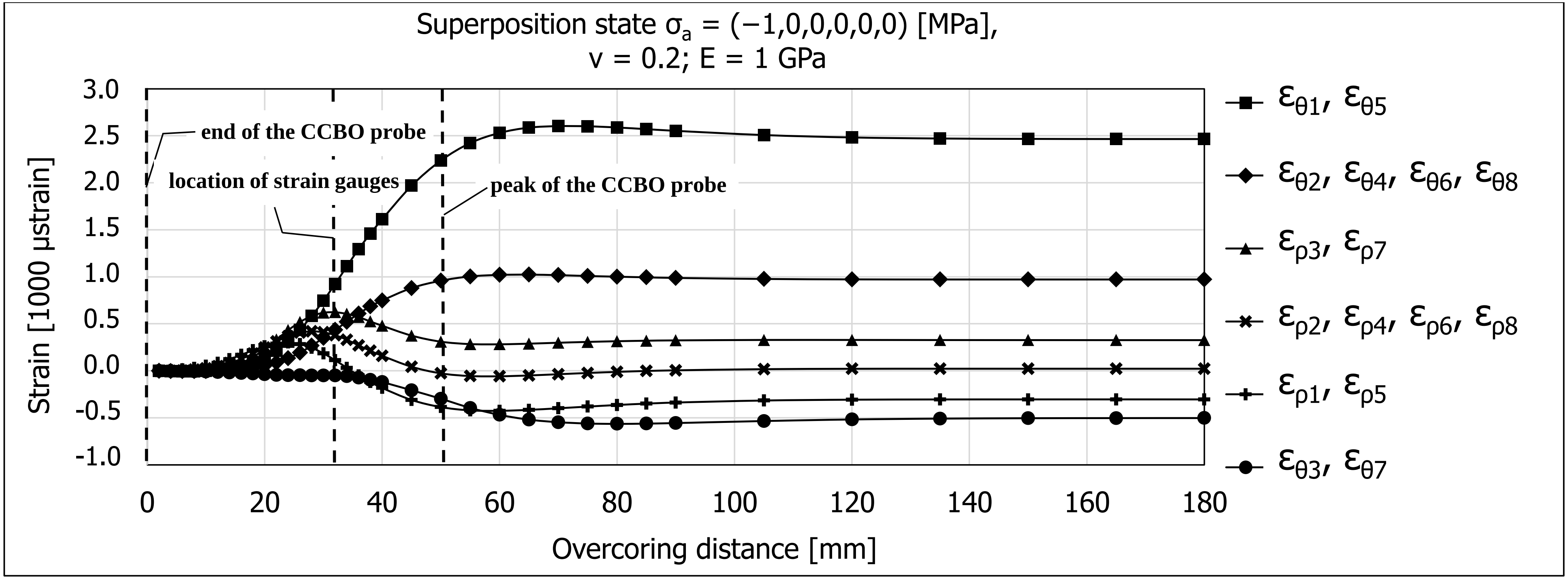

***Fig. 2** Numerical model output of responding strains for the CCBO probe positioned at the borehole end and subsequently overcored, in this case equipped with 8 nests distributed around its circumference. Each nest accommodates strain gauges measuring strains in the tangential (ε_θ) and longitudinal (ε_ρ) directions*

2 Parametric distribution matrix

Fig. 2 shows the strain development during overcoring, as obtained from the numerical model. The results correspond to a probe with eight nests distributed along the perimeter of the conical probe, with two directions considered in each nest: the tangential direction corresponding to strain ε_θ, and the longitudinal direction corresponding to strain ε_ρ.

The strain curves exhibit characteristic inflection points during overcoring in the immediate vicinity of the strain gauge locations on the conical probe, followed by a stabilization of strain. When the stabilized strains are recorded for individual gauges as a function of their position along the probe perimeter, the corresponding functional relationships of strain around the conical probe are derived and assembled into the distribution matrix (Eq. 4). Most of these dependencies can be represented as composite trigonometric functions. Some, however, exhibit a relatively constant behaviour, for example in the case of the superposition state σ_c, which represents the activation of a unit stress in the longitudinal axis of the conical probe (the Z-axis in Fig. 1).

$$\begin{pmatrix}\varepsilon_\theta\\ \varepsilon_\rho\\ \varepsilon_\xi\end{pmatrix}=\begin{pmatrix} T_{xx1}-T_{xx2}\cos 2\theta & T_{yy1}+T_{yy2}\cos 2\theta & T_{zz} & T_{xy}\sin 2\theta & T_{yz}(-\sin\theta) & T_{xz}\cos\theta\\ L_{xx1}+L_{xx2}\cos 2\theta & L_{yy1}-L_{yy2}\cos 2\theta & L_{zz} & L_{xy}(-\sin 2\theta) & L_{yz}(-\sin\theta) & L_{xz}\cos\theta\\ P_{xx1}+P_{xx2}\left(-\cos 2\left(\theta-\frac{\pi}{8}\right)\right) & P_{yy1}+P_{yy2}\cos 2\left(\theta-\frac{\pi}{8}\right) & P_{zz} & P_{xy}\sin 2\left(\theta-\frac{\pi}{8}\right) & P_{yz}\left(-\cos\left(\theta-\frac{\pi}{8}\right)\right) & P_{xz}\left(-\sin\left(\theta-\frac{\pi}{8}\right)\right)\end{pmatrix}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{pmatrix}\cdot\frac{1}{E} \qquad (4)$$

In Eq. 4, the parametric distribution matrix is described using functions, where each function is composed of the parametric coefficients from Tab. 1 and 2, which vary depending on the Poisson's ratio. The parametric coefficients are deliberately denoted with indices reflecting their origin. For example, the element located in the first row and first column of the distribution matrix in Eq. 4 is composed of the coefficients T_{xx1} and T_{xx2}. The symbol T indicates that the term corresponds to strain in the tangential direction ε_θ (see Fig. 1). The subscript xx denotes that these coefficients were obtained under the superposition state σ_a, corresponding to the application of stress in the X direction. Two coefficients are included in this specific element because two terms are necessary to describe the trigonometric function of tangential strain with respect to the circumferential position around the CCBO probe. Similarly, the element located in the second row and third column of the distribution matrix is represented by the coefficient L_{zz}. This indicates that the coefficient is associated with the superposition state σ_c, corresponding to the activation of stress in the Z direction, and describes the variation of longitudinal strain, ε_ρ, along the circumference of the conical probe. Although all those coefficients of the parametric distribution matrix in Eq. 4 are obtained numerically, their determination through this study demonstrates the inherent analytical nature of the distribution matrix.

The parametric coefficients of the individual components of the distribution matrix were estimated using the method of least squares. For each parameter, both the point estimate and its standard error (SE) are reported (in Tab. 1–4). The standard error is a quantitative metric that quantifies the uncertainty of the parameter estimate, and corresponds to the standard deviation of its sampling distribution. A smaller standard error is indicative of a higher precision of the estimate.

Tab. 1 – Parametric distribution coefficients T and L [1000 µstrain]

	$T_{xx1} = T_{yy1}$	$T_{xx2} = T_{yy2}$	$L_{xx1} = L_{yy1}$	$L_{xx2} = L_{yy2}$	T_{zz}	L_{zz}	$T_{yz} = T_{xz}$	$L_{yz} = L_{xz}$	T_{xy}	L_{xy}
$\nu = 0.1$	0.9934	1.6031	0.1105	0.3277	−0.1402	0.6014	0.0679	1.5372	3.24	0.650
$\nu = 0.2$	0.9888	1.6015	0.0244	0.3496	−0.2527	0.5797	0.0777	1.6435	3.23	0.690
$\nu = 0.25$	0.9836	1.5873	0.0190	0.3583	−0.3118	0.5700	0.0825	1.6928	3.20	0.705
$\nu = 0.3$	0.9776	1.5630	−0.0625	0.3649	−0.3747	0.5607	0.0870	1.7389	3.15	0.715
$\nu = 0.4$	0.9539	1.4782	−0.1485	0.3682	−0.5340	0.5413	0.0927	1.8291	2.95	0.710

Tab. 2 – Parametric distribution coefficient P [1000 µstrain]

	$P_{xx1} = P_{yy1}$	$P_{xx2} = P_{yy2}$	P_{zz}	$P_{yz} = P_{xz}$	P_{xy}
$\nu = 0.1$	0.61	1.09	0.18	1.78	2.21
$\nu = 0.2$	0.59	1.11	0.11	1.91	2.20
$\nu = 0.25$	0.55	1.09	0.07	1.98	2.16
$\nu = 0.3$	0.53	1.08	0.03	2.04	2.15
$\nu = 0.4$	0.48	1.04	−0.07	2.14	2.00

Tab. 3 – Standard error (SE) of the parametric distribution coefficients T and L [–]

	$T_{xx1} = T_{yy1}$	$T_{xx2} = T_{yy2}$	$L_{xx1} = L_{yy1}$	$L_{xx2} = L_{yy2}$	T_{zz}	L_{zz}	$T_{yz} = T_{xz}$	$L_{yz} = L_{xz}$	T_{xy}	L_{xy}
SE	0.000711	0.001005	0.000689	0.000976	0.000476	0.000919	0.000283	0.001294	0.005166	0.000965

Tab. 4 – Standard error (SE) of the parametric distribution coefficient P [–]

	$P_{xx1} = P_{yy1}$	$P_{xx2} = P_{yy2}$	P_{zz}	$P_{yz} = P_{xz}$	P_{xy}
SE	0.00069	0.000976	0.000676	0.003345	0.001615

It is evident from the results obtained that the estimated parametric distribution coefficients exhibit low standard errors, thus substantiating their stability and reliability. This finding suggests that the individual coefficients of the distribution matrix have been determined with sufficient accuracy, providing a robust foundation for further interpretation and analysis of the stress tensor determination.

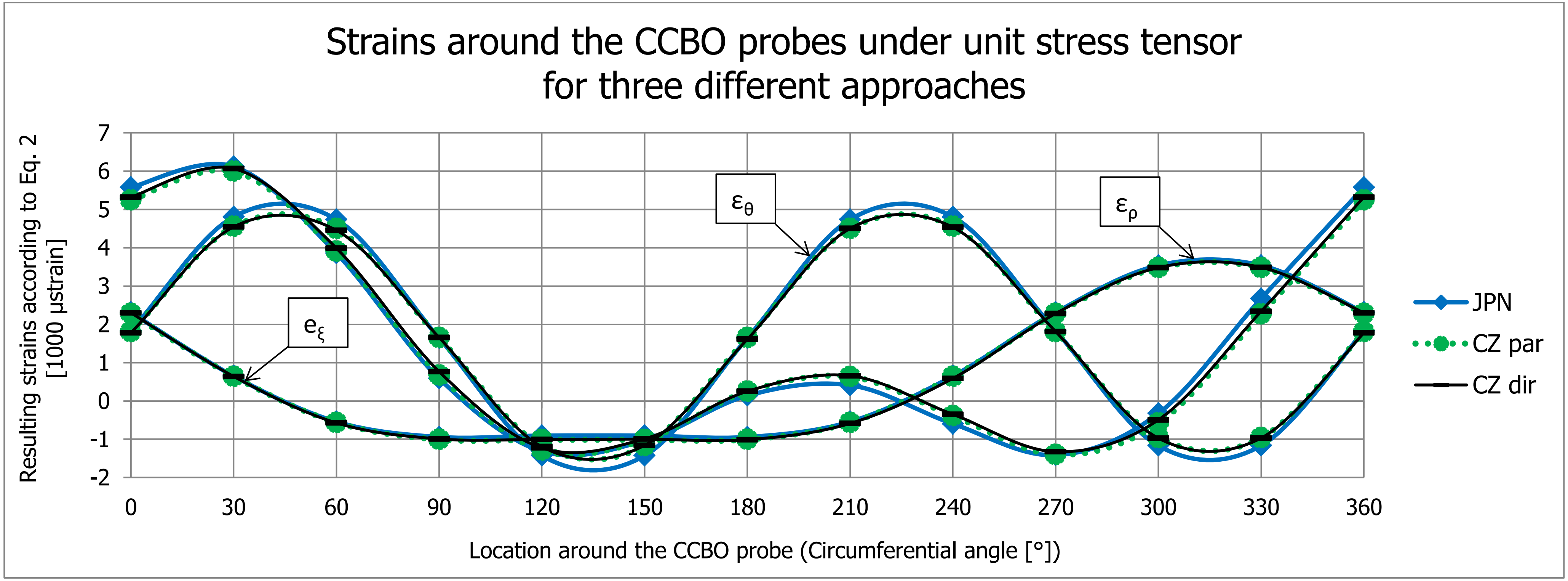

Fig. 3 Strains determined from Eq. 2 using the superposition states from Eq. 3 for three different approaches: JPN – the original Japanese approach; CZ par – using the parametric distribution matrix from Eq. 4; CZ dir – using direct substitution of strains obtained from the numerical model into Eq. 2

Although the estimated coefficients appear to be determined with high precision, this apparent accuracy should be interpreted with caution. A useful analogy can be drawn from laboratory testing of rock samples under standardised conditions: even when experiments are conducted according to the same protocol in another laboratory, the results may differ. In addition, variations are an inherent feature of the conception of numerical models.

For instance, a comparison of the strains calculated from Eq. 1 presented in this paper with those obtained using the Japanese version of the distribution matrix – assuming a unit stress tensor from Eq. 3 – reveals noticeable deviations, as illustrated in Fig. 3. This does not imply that any of the compared distribution matrices is inherently less accurate; rather, it highlights that the presented coefficients of the distribution matrix may be subject to larger uncertainties than suggested by the standard errors of the individual estimates. Indeed, if multiple numerical models were developed using different software platforms, the resulting coefficients would likely exhibit a spread of values, reflecting the inherent variability in the modelling process.

In addition to the parametric distribution matrix, the results obtained from the direct distribution matrix are also presented for comparison. Accordingly, the figure presents a comparison of three approaches: the original Japanese results (JPN; Sugawara & Obara, 1999), the Czech parametric distribution matrix (CZ par), and the Czech direct distribution matrix (CZ dir). For this comparison, the Japanese results were adapted to the Czech convention, accounting for differences in axis orientation and tensor component ordering. No relative deviations are presented in the figure to avoid potential misunderstandings that could arise from extreme values; due to the trigonometric nature of the strain distributions, representing the results as relative deviations would be misleading. Instead, the figure illustrates the resulting strains in the directions of ε_θ, ε_ρ, and ε_ξ, calculated under the assumption of a unit stress tensor and a unit elastic modulus, at a total of twelve positions around the circumference of the conical probe. The presented results of these three outputs demonstrate a close agreement among the methods, with very similar strain distributions observed along the full perimeter of the probe.

3 Conclusion

The primary objective of this study was to systematically present the standard errors of the estimated distribution matrix coefficients of the CCBO method, which are essential for converting measured strains on a conical probe into the corresponding stress tensor in a rock mass. It is important to acknowledge that these standard errors were determined under the specific reference conditions of a unit stress tensor of 1 MPa and a Young's modulus of 1 GPa, under which the coefficients are expressed in units of 1000 μstrain. For isotropic materials, the distribution matrix can be derived in the form of functional dependencies of strain along the probe circumference for a set of selected Poisson's ratio values. Alternatively, the distribution matrix can be obtained by extracting strains from a numerical model and incorporating them directly into the computational procedure through Eq. 2. Both methodologies were implemented and systematically compared with the original approach proposed by Japanese researchers.

As the anisotropic stiffness matrix involves multiple independent elastic moduli, factoring out a single modulus is not feasible, and a distribution matrix parameterized solely by the rock's deformation parameters cannot be established. It would, however, be possible to formulate a parametric distribution matrix under a transversely isotropic constitutive model, in which one elastic modulus is factored out while the other is held constant with the remaining parameters adjusted to satisfy the constitutive admissibility constraints. Nevertheless, given the computational effort and algebraic complexity, a more effective approach for anisotropic media is to construct a direct distribution matrix by extracting the simulated strains directly from the numerical model, so that the matrix consists of numerical values rather than approximate, numerically determined functional relations that are not available in closed form. In practical computations, these choices lead to differences in the resulting stress tensor on the order of single-digit to low tens of pascals when comparing the Czech parametric distribution matrix (CZ par) with the Czech direct distribution matrix (CZ dir). By contrast, when the Czech parametric distribution matrix (CZ par) is compared with the parametric distribution matrix derived from the original Japanese solution (JPN), the deviations are larger and can reach the low hundreds of pascals (under the specific reference conditions of a unit stress tensor of 1 MPa and a Young's modulus of 1 GPa).

The comparative analysis of the three distribution matrices – the original Japanese matrix (JPN), the Czech parametric distribution matrix (CZ par), and the Czech direct distribution matrix (CZ dir) – demonstrates that all methods yield highly consistent strain distributions for the considered stress tensor. While minor deviations are observed at specific locations along the probe circumference, the methods exhibit a high degree of overall concordance, indicating that both the parametric and direct approaches are capable of reliably reproducing the strain field. This observed consistency substantiates the applicability of the proposed Czech parametric distribution matrix, while the residual discrepancies are most likely attributable to inherent modelling uncertainties rather than to systematic methodological errors.

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Authors:
[1]Ing. Alice Petrlíková, Ph.D. - Institute of Geonics CAS, Studentská 1768, 708 00 Ostrava-Poruba, Czech Republic, alice.petrlikova@ugn.cas.cz
[2]RNDr. Lubomír Staš, CSc. - Institute of Geonics CAS, Ústav geoniky AV ČR, v. v. i., Studentská 1768, 708 00 Ostrava-Poruba, Czech Republic
[3]Doc. Ing. Petr Koníček, Ph.D. - Institute of Geonics CAS, Ústav geoniky AV ČR, v. v. i., Studentská 1768, 708 00 Ostrava-Poruba, Czech Republic